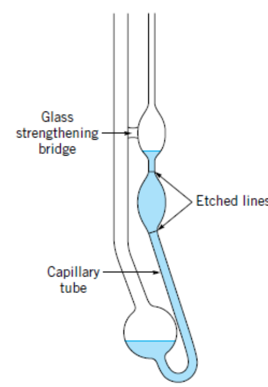
ENGR207 Fluid Mechanics

Assignment #1: Introduction to Fluid Properties

For any missing viscosity values, use the chart at the end of the assignment.

1. (P1.25) For a certain liquid a hydrometer (a device for measuring viscosity) reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight?
2. (P1.27+) Estimate the number of kilograms of mercury it would take to fill your bath tub. List all assumptions and show all calculations.
3. (P1.28) A liquid when poured into a graduated cylinder is found to weigh 8 N when occupying a volume of 500 ml (milliliters). Determine its specific weight, density, and specific gravity.
4. (P1.29)The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20°C. (see chart at the end of the assignment)
5. (P1.41) For the capillary-tube viscometer shown, the liquid to be tested is drawn into the tube to the level of the top etched line. The time is then obtained for the liquid to drain to the bottom etched line. The kinematic viscosity, ν, in m^2/s is then obtained from the equation $\nu = KR^4t$ where K is a constant, R is the radius of the capillary tube in mm, and *t* is the drain time in seconds. When glycerin, whose viscosity is 1.4 Pa.s at 20°C and density is 1260 kg/m^3, is used as a calibration fluid in a particular viscometer the drain time is 1,430 s. When a liquid having a density of 970 kg/m^3 is tested in the same viscometer the drain time is 900 s. What is the dynamic viscosity of this liquid?



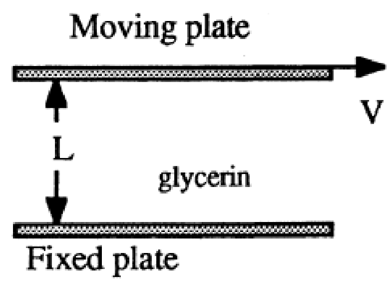
6. (P1.55) The viscosity of blood is to be determined by measurements of shear stress, τ, and rate of shearing strain, *du/dy*, obtained from a small blood sample tested in a suitable viscometer. Based on the data given below, determine if the blood is a Newtonian or non-Newtonian fluid. Explain how you arrived at your answer.

du/dy (s^{-1})	0.04	0.06	0.12	0.18	.30	0.52	1.12	2.1
τ (N/m^2)	2.25	4.5	11.25	22.5	45	90	225	450

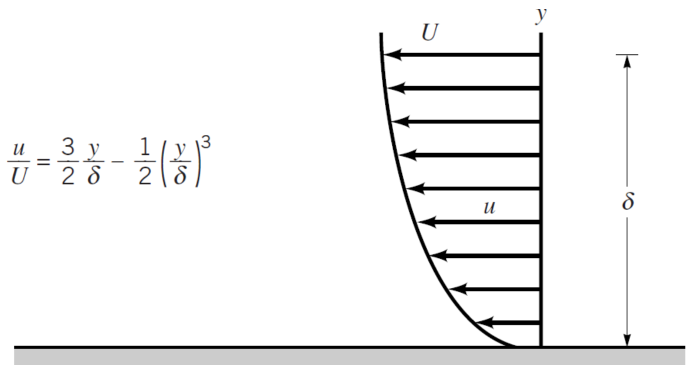
7. (P1.46) Some experimental data obtained for a particular non-Newtonian fluid at 25°C are shown below. Plot these data and fit a second-order polynomial to the data using a suitable graphing program. What is the apparent viscosity of this fluid when the rate of shearing strain is 70 s^{-1}. Is this apparent viscosity larger or smaller than that for water at the same temperature?

γ (s^{-1})	0	50	100	150	200
τ (Pa)	0	100.8	373.8	884.3	1515.3

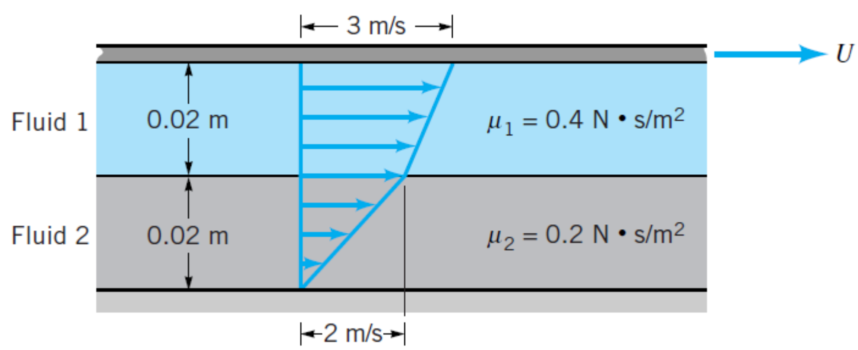
8. In the figure shown, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s?



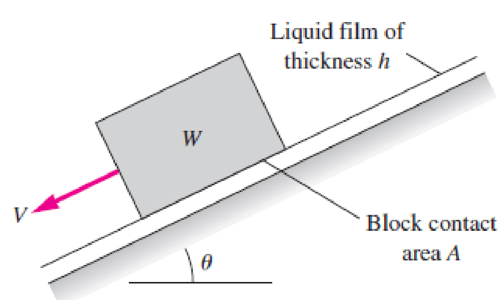
9. (P1.58) A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of 4×10^{-4} m^2/s flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero, and the velocity profile near the surface is shown in the figure on the right. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



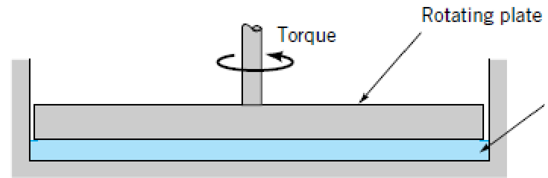
10. (P1.54) Let two layers of fluid be dragged along by the motion of an upper plate as shown in the figure below. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the bottom plate. Determine the ratio of these two shear stresses.



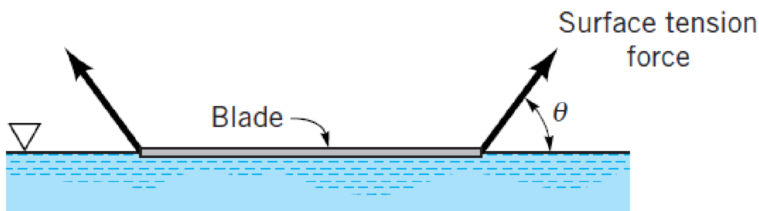
11. A block of weight W slides down an inclined plane while lubricated by a thin film of oil, as in the figure below. The film contact area is A and its thickness is h. Assuming a linear velocity distribution in the film, derive an expression for the "terminal" (zero-acceleration) velocity V of the block. Find the terminal velocity of the block if the block mass is 6 kg, A=35 cm^2, θ=15°, and the film is 1-mm-thick SAE 30 oil at 20°C (μ=0.4 Pa.s).



12. (P1.65) A 30 cm diameter circular plate is placed over a fixed bottom plate with a 2.5 mm gap between the two plates filled with glycerin as shown in the figure below. Determine the torque required to rotate the circular plate slowly at 2 rpm. Assume that the velocity distribution in the gap is linear and that the shear stress on the edge of the rotating plate is negligible.

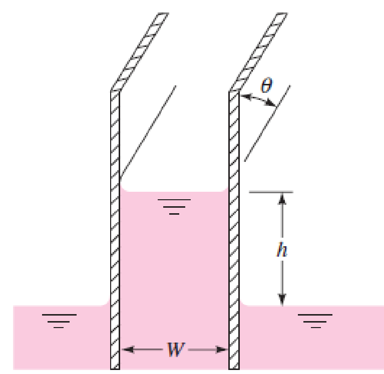


13. (P1.84) For a razor blade resting on the surface of water, assume that the surface tension forces act at an angle θ relative to the water surface as shown in the figure below. (a) The mass of the double edge blade is 0.64 gm , and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. (b) The mass of the single-edge blade is 2.6 gm, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations. Surface tension of water is 0.073 N/m.



14. (P1.82) Estimate the excess pressure inside a raindrop having a diameter of 3 mm.
15. Estimate the excess pressure inside a soap bubble having a diameter of 3 mm.

16. Derive an expression for the capillary height change h for a fluid of surface tension σ and contact angle θ between two vertical parallel plates a distance W apart, as in the figure shown. What will *h* be for water at 20°C if W = 0.5 mm? Assume the contact angle is 30°.



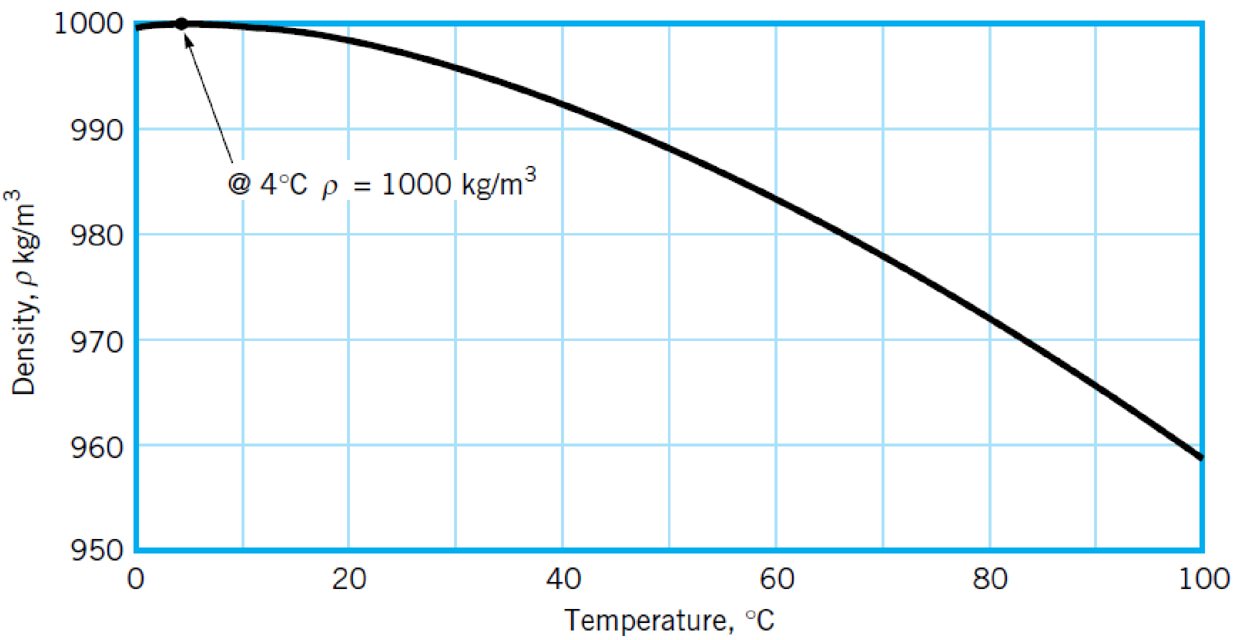
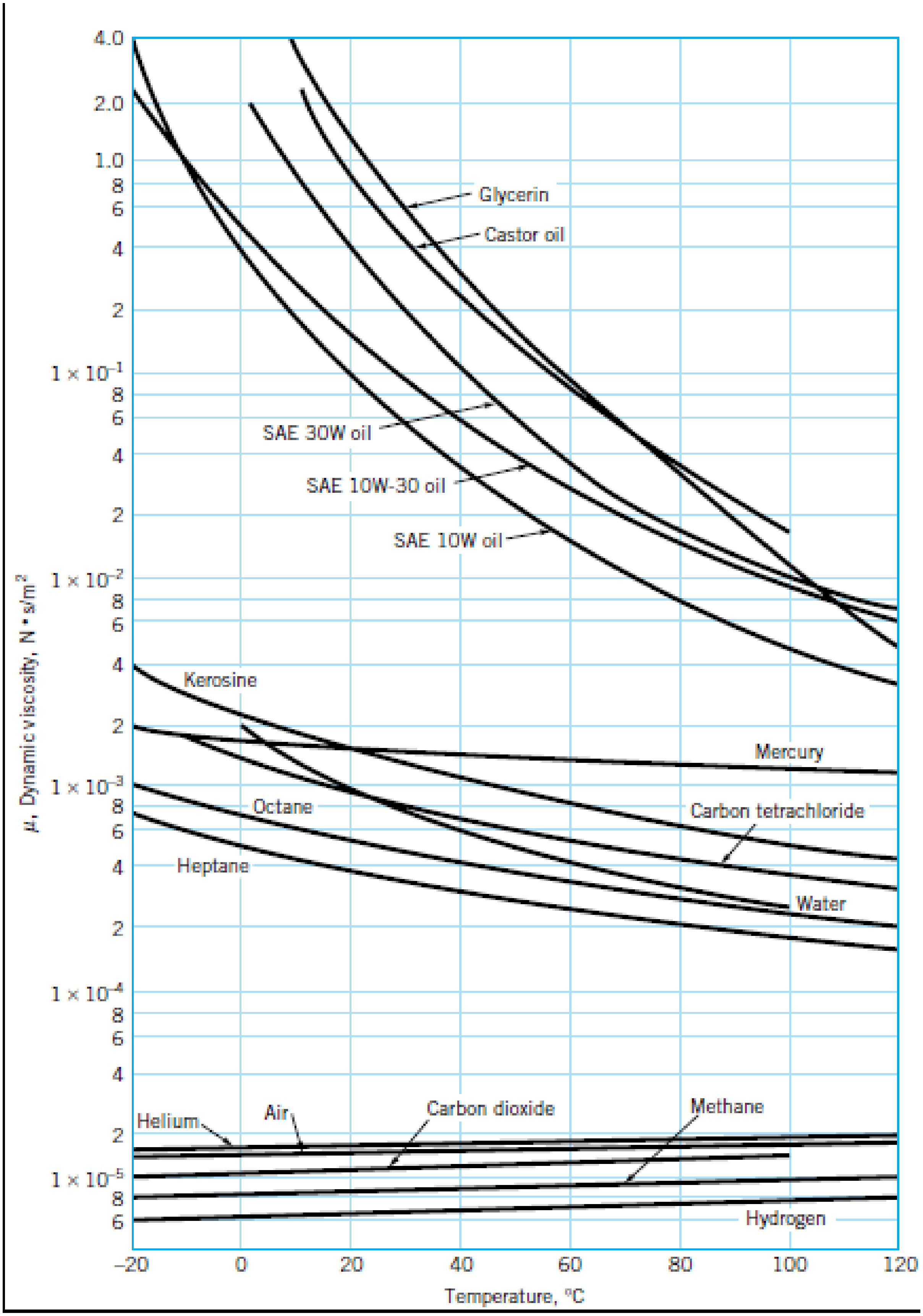


Figure 1 Density of water as a function of temperature


Glycerin
Castor oil
SAE 30W oil
SAE 10W-30 oil
SAE 10W oil
Kerosine
Mercury
Carbon tetrachloride
Octane
Heptane
Water
Helium
Air
Carbon dioxide
Methane
Hydrogen
μ, Dynamic viscosity, N • s/m²
Temperature, °C
4.0
2.0
1.0
8
6
4
2
1 × 10⁻¹
1 × 10⁻²
1 × 10⁻³
1 × 10⁻⁴
1 × 10⁻⁵
–20
0
20
40
60
80
100
120

ENGR 207 Fluid Mechanics

Solution of Assignment #1

1) $SG = 1.15$

$\therefore \rho = SG \times \rho_{H_2O \text{ at } 4^\circ C}$

$= 1.15 \times 1000 = \boxed{1150 \text{ kg/m}^3}$

Specific weight $= \rho g$

$= 1150 \times 9.8 = \boxed{11270 \text{ N/m}^3}$

2) assume bathtub dimensions are $1.5 \times 0.6 \times 0.7 \text{ m}^3$

$\therefore$ Volume $= 1.5 \times 0.6 \times 0.7 = 0.63 \text{ m}^3$

from tables $\rho_{mercury} = 13594 \text{ kg/m}^3$

$\therefore m = \rho V = 0.63 \times 13594 = \boxed{8564.22 \text{ kg}}$

3) Volume $= 0.5 \text{ L} = 0.5 \times 10^{-3} \text{ m}^3$ $\quad$ $W = mg = 8 \text{ N}$

Specific weight $= \dfrac{W}{\text{Volume}} = \dfrac{8}{0.5 \times 10^{-3}} = \boxed{16000 \text{ N/m}^3}$

$\therefore \rho = \dfrac{\text{Specific weigh}}{g} = \dfrac{16000}{9.8} = \boxed{1632.65 \text{ kg/m}^3}$

$SG = \dfrac{\rho}{\rho_{H_2O}} = \dfrac{1632.65}{1000} = \boxed{1.632}$

4) Mass of liquid inside pip $= 0.369 - \frac{0.153}{9.8} = 0.353$ kg

$$\rho = \frac{m}{V} = \frac{0.353}{0.355\times10^{-3}} = \boxed{995.46 \ kg/m^3}$$

∴ Density of Pop is almost the same like density of water at 20°c ($\rho_{H_2O-20°c} = 998 \ kg/m^3$)

$$SG = \frac{995.46}{1000} = \boxed{0.995}$$

Specific weight $= \rho g = 995.46 \times 9.8 = \boxed{9755.5 \ N/m^3}$

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5) $\nu = k R^4 t \longrightarrow \rho_{glycerin} = 1260 \ kg/m^3$ missing in sheet

For glycerin: $\frac{1.4}{1260} = k R^4 \times 1430$

$\therefore k R^4 = 7.77\times10^{-7} \quad m^2/s^2$

$\nu_{liquid} = k R^4 \times 900$

$= 7.77\times10^{-7}\times900 = 6.99\times10^{-4} \ m^2/s$

$\mu = \nu\rho = 6.99\times10^{-4}\times970 = \boxed{0.678 \ Pa.s}$

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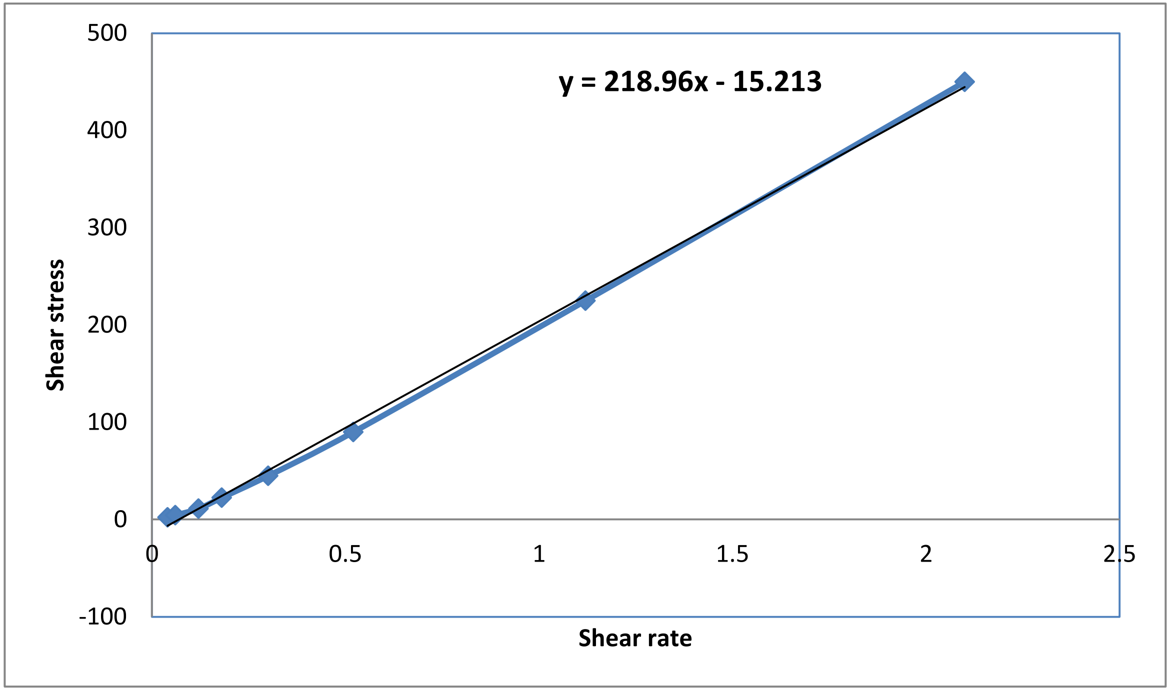
6) When you draw the τ vs $\frac{du}{dy}$ curve, you will find it is almost linear (see next page).

∴ Blood is a Newtonian liquid.

du/dy (s^{-1})	0.04	0.06	0.12	0.18	0.3	0.52	1.12	2.1
τ (N/m^2)	2.25	4.5	11.25	22.5	45	90	225	450



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7) The figure below shows a plot of the shear stress vs shear rate. The trend line gives the following equation:

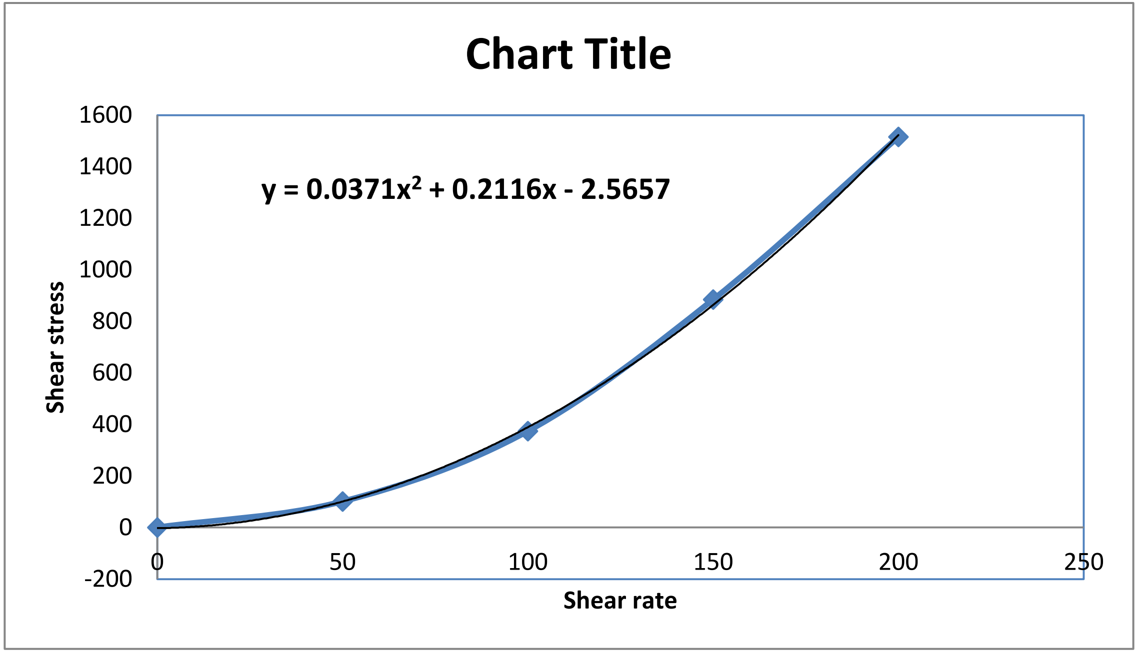
$$\tau = 0.0371\dot{\gamma}^2 + 0.2116\dot{\gamma} - 2.5657$$

$$\mu = \frac{d\tau}{d\dot{\gamma}} = 0.0742\dot{\gamma} + 0.2116$$

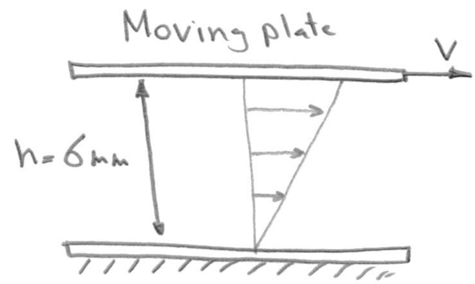
$\therefore$ at $\dot{\gamma} = 70\ s^{-1}$ $\longrightarrow$ $\tau = 0.0742(70) + 0.2116$

$= \boxed{5.4\ Pa.s}$

γ (s^{-1})	0	50	100	150	200
τ (Pa)	0	100.8	373.8	884.3	1515.3



8) $\tau = \mu \dfrac{du}{dy}$



for glycerin at 20°C $\longrightarrow \mu = 1.4$ Pa.s

$\therefore \dfrac{du}{dy} = \dfrac{V}{h} = \dfrac{5.5}{0.006} = 916.67\ s^{-1}$

$\therefore \tau = \mu \dfrac{du}{dy} = 1.4 \times 916.67 = \boxed{1283.3\ \text{Pa}}$

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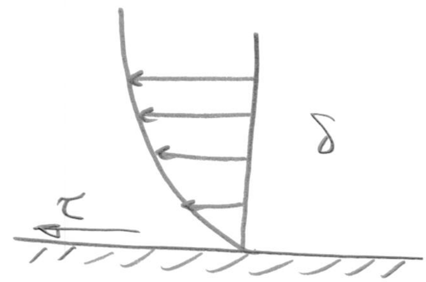
9) $\dfrac{u}{U} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^2$



$\dfrac{du}{dy} = \left[\dfrac{3}{2}\left(\dfrac{1}{\delta}\right) - \dfrac{y}{\delta^2}\right] \times U$

on the surface $y=0 \longrightarrow \therefore \boxed{\dfrac{du}{dy} = \dfrac{3U}{2\delta}}$

$\therefore \tau = \mu \dfrac{du}{dy}$

$\mu = \nu \rho = 4 \times 10^{-4} \times 0.92 \times 1000 = 0.368$ Pa.s

$\therefore \tau = 0.368 \times \dfrac{3U}{2\delta} = \boxed{0.552\ \dfrac{U}{\delta}}$

* shearing stress of the fluid on the surface will be directed <u>towards the left</u>

10) for the bottom plate:

$$\frac{du}{dy} = \frac{U}{b} = \frac{2}{0.02} = 100\ s^{-1}$$

$$\therefore \tau_2 = \mu_2 \left.\frac{du}{dy}\right|_2 = 0.2 \times 100 = \boxed{20\ Pa}$$

for the top plate:

$$\frac{du}{dy} = \frac{\Delta U}{b} = \frac{3-2}{0.02} = 50\ s^{-1}$$

$$\therefore \tau_1 = \mu_1 \frac{du}{dy_1} = 0.4 \times 50 = \boxed{20\ Pa}$$

$$\therefore \boxed{\frac{\tau_1}{\tau_2} = \frac{20}{20} = 1}$$

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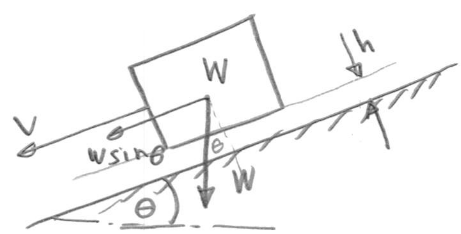
11) at terminal velocity (zero acceleration):

Viscous friction force = component of block weight along plane



$$\therefore \tau A = W \sin\theta$$

$$\therefore \mu A \frac{du}{dy} = W \sin\theta$$

but $\frac{du}{dy} = \frac{V}{h}$

$$\therefore \mu A \frac{V}{h} = W \sin\theta$$

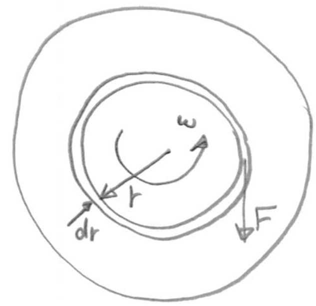
$$\therefore \boxed{V = \frac{h W \sin\theta}{\mu A}}$$

12) Since velocity of any point on the plate is function of radius (r), then shear stress will vary from zero at center of plate to maximum at the tip.

$$V = \omega r$$

$$\frac{du}{dy} = \frac{V}{h} = \frac{\omega r}{h}$$

$$\tau = \mu \frac{du}{dr} = \mu \frac{\omega r}{h} = f(r)$$



$$\therefore \; dT = r \cdot dF$$

$$= r\,\tau\, dA = r\,\frac{\mu \omega r}{h}\, 2\pi r\, dr$$

$$= \frac{2\pi \mu \omega}{h} r^3 dr$$

$$\therefore \; \therefore T = \int_0^R \frac{2\pi \mu \omega}{h} r^3 dr$$

$$= \frac{2\pi \mu \omega}{h}\left[\frac{r^4}{4}\right]_0^R = \frac{\pi \mu \omega R^4}{2h}$$

$\mu = 1.4$ Pa

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2}{60} = 0.209 \text{ rad/s}$$

$$\therefore T = \frac{\pi \times 1.4 \times 0.209 \times (0.15)^4}{0.0025}$$

$$\boxed{T = 0.186 \text{ N.m}}$$

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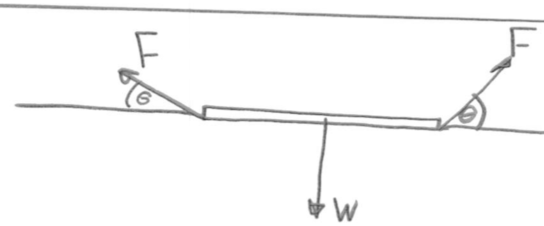
13) $W = F\sin\theta$



$$\therefore \sin\theta = \frac{W}{F} = \frac{mg}{\sigma L}$$

$$= \frac{0.64 \times 10^{-3} \times 9.8}{0.073 \times 0.208} = 0.417$$

$$\therefore \boxed{\theta = 24.65^\circ}$$

b) for the single blade: $F\sin\theta = \sigma L \sin\theta = 0.073 \times 0.154 \times \sin 24.65$

$$= 4.68 \times 10^{-3} \text{ N}$$

$W = mg = 2.6 \times 10^{-3} \times 9.8 = 25 \times 10^{-3}$ N

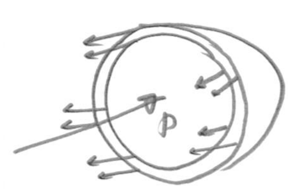
14) For a droplet:

$$\Delta P \, \pi r^2 = 2\pi r \sigma$$

$$\therefore \Delta P = \frac{2\sigma}{r}$$

$$= \frac{2 \times 0.073}{0.003} = \boxed{48.67 \text{ Pa}}$$

15) For a soap bubble, there are two liquid-air interfaces:

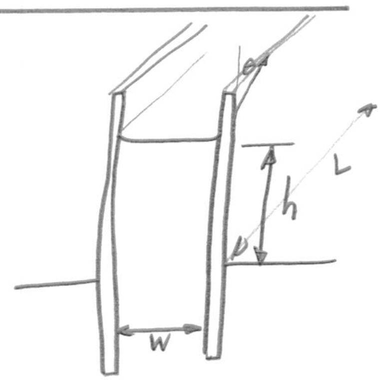


$$\therefore \Delta P \, \pi r^2 = 2 \times 2\pi r \sigma$$

$$\therefore \Delta P = \frac{4\sigma}{r} = \frac{4 \times 0.073}{0.003}$$

$$= \boxed{97.33 \text{ Pa}}$$

16) For equilibrium of the liquid column:



Weight = Surface tension force

$$\therefore W \times h \times \cancel{L} \times \rho g = 2\sigma \cancel{L} \cos\theta$$

$$\therefore \boxed{h = \frac{2\sigma \cos\theta}{W \rho g}}$$

$$\therefore h = \frac{2 \times 0.073 \cos 30}{0.5 \times 10^{-3} \times 1000 \times 9.8} = 0.0258 \text{ m}$$

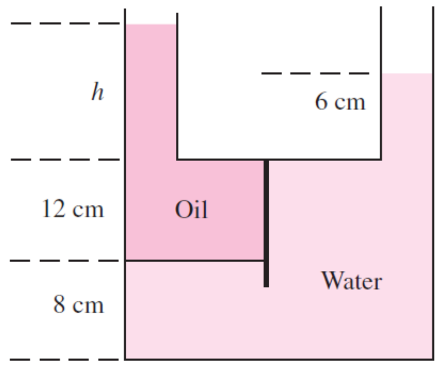
$$\boxed{\therefore h = 25.8 \text{ mm}}$$

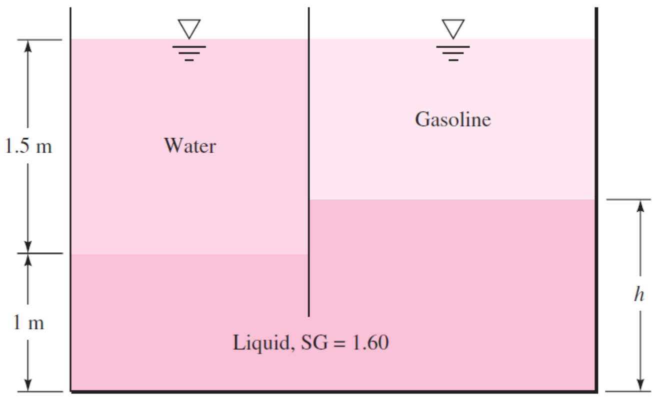
M201 Hydraulics

Assignment #2: Hydrostatics

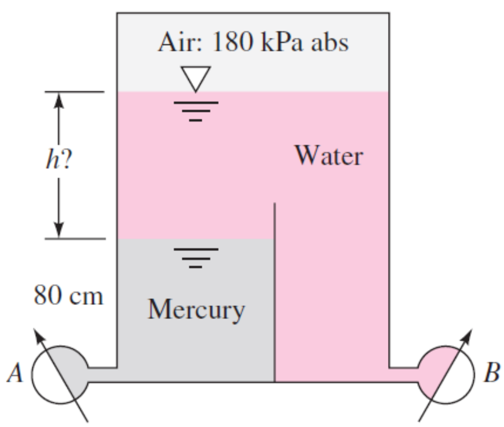
For any missing viscosity values, use the chart at the end of the assignment.

1. The tank shown on the right contains water and immiscible oil at 20 C. What is h in cm if the density of the oil is 898 kg/m^3?

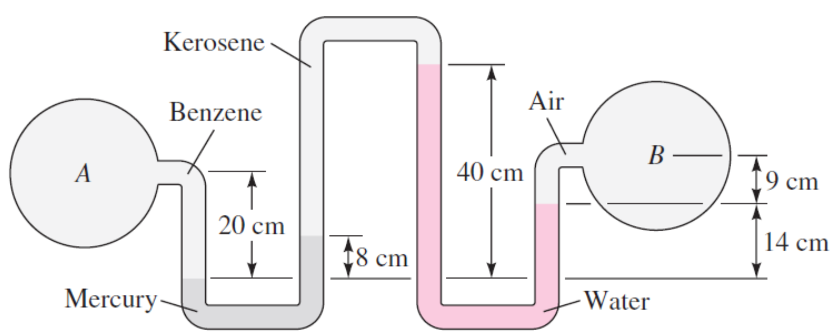




2. The 20C water and gasoline surfaces in the figure on the left are open to the atmosphere and at the same elevation. What is the height h of the third liquid in the right leg? Specific weight of gasoline is 6670 N/m.



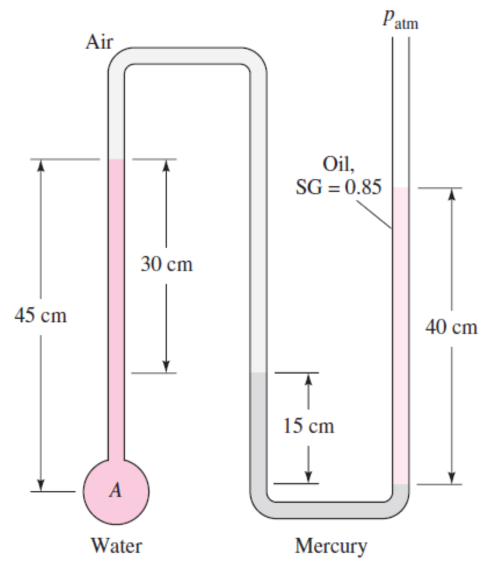
3. At 20C gage A reads 350 kPa absolute. What is the height h of the water in cm? What should gage B read in kPa absolute? Density of mercury is 13580 kg/m^3.

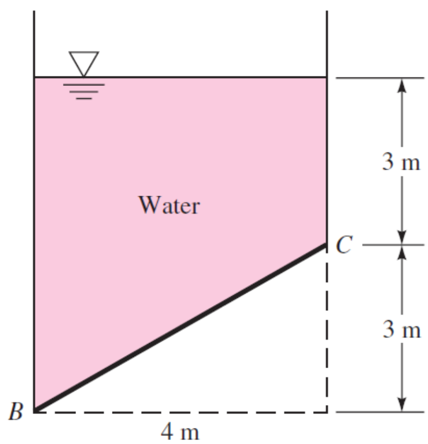


4. In the figure on the left, all fluids are at 20C. Determine the pressure difference between points A and B. Specific weights are:

Benzene:	8640 N/m^3	Mercury:	133100 N/m^3
Kerosene:	7885 N/m^3	Water:	9790 N/m^3

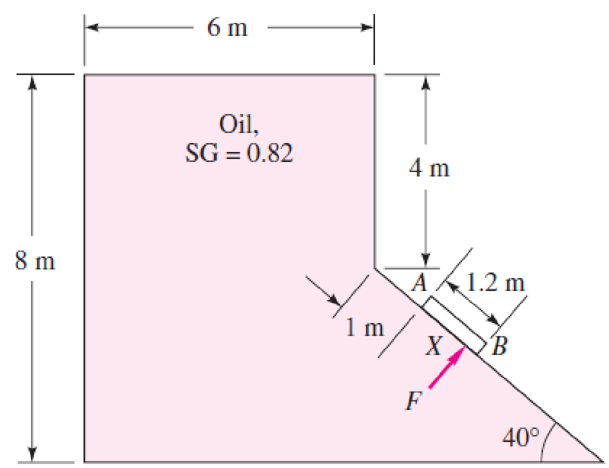
5. In the figure on the right, determine the gage pressure at point A. Is it higher or lower than atmospheric?

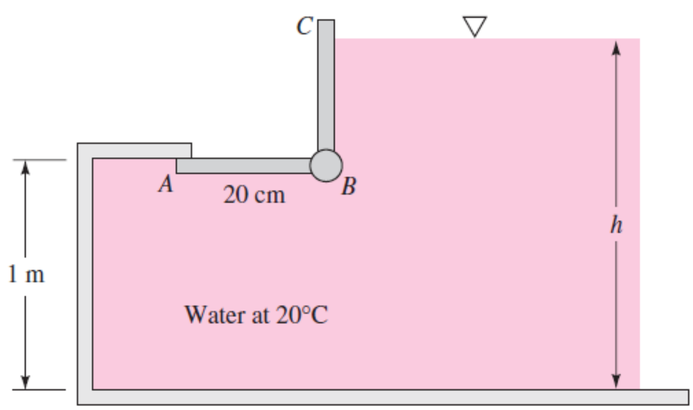




6. The tank in the figure on the left is 2 m wide into the paper. Neglecting atmospheric pressure, find the resultant hydrostatic force on the surface BC.

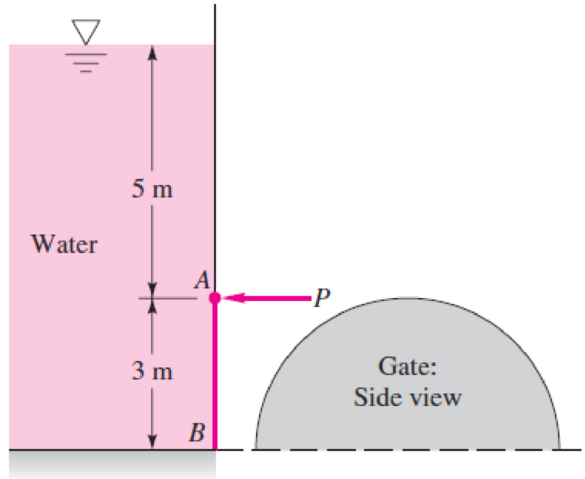
7. Gate AB in the figure on the right is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric pressure, compute the force F on the gate and its center-of-pressure position X.



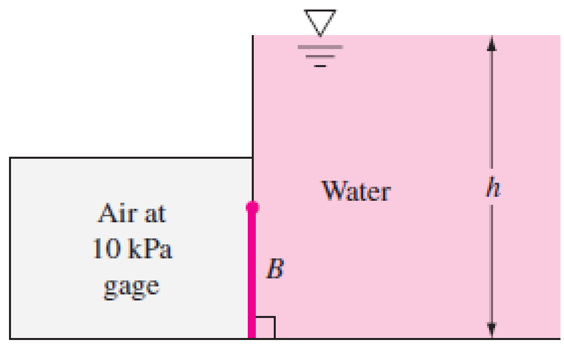


8. Gate ABC in the figure on the left has a fixed hinge line at B and is 2 m wide into the paper. The gate will open at A to release water if the water depth is high enough. Compute the depth h for which the gate will begin to open. Neglect the weight of the gate itself.

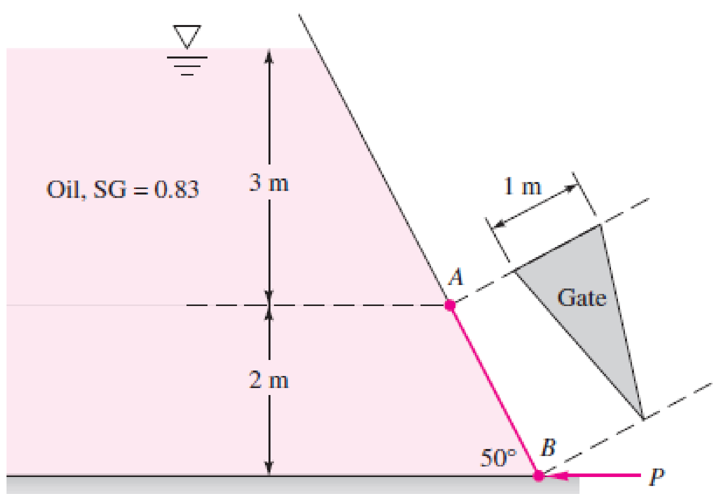
9. Gate AB in the Figure on the left is semicircular, hinged at B, and held by a horizontal force P at A. What force P is required for equilibrium?



10. Gate B in the figure below is 30 cm high, 60 cm wide into the paper, and hinged at the top. What water depth h will first cause the gate to open?



11. Isosceles triangle gate AB in the figure below is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?



M201 Hydraulics

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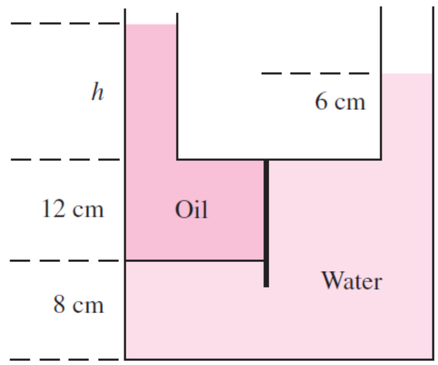
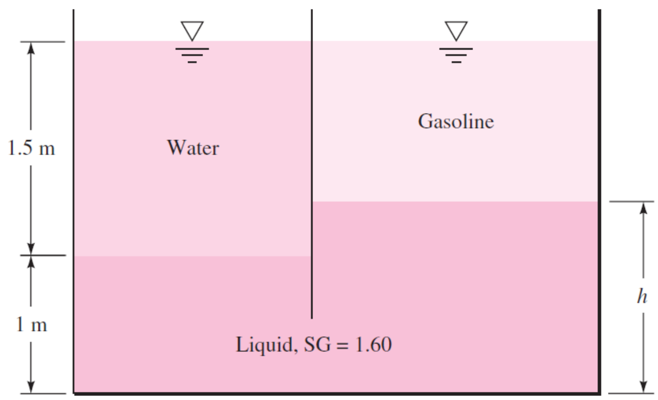
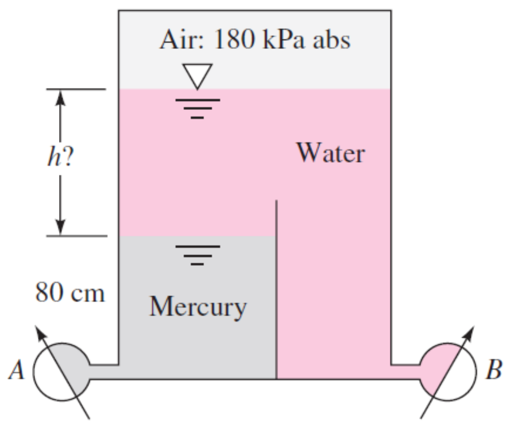
1. The tank shown on the right contains water and immiscible oil at 20 C. What is h in cm if the density of the oil is 898 kg/m^3?

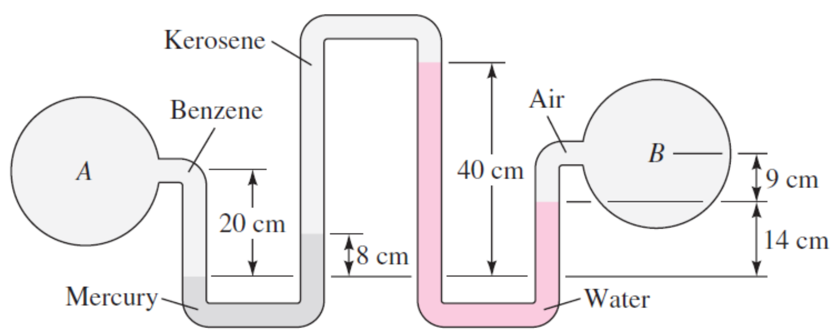




2. The 20C water and gasoline surfaces in the figure on the left are open to the atmosphere and at the same elevation. What is the height h of the third liquid in the right leg? Specific weight of gasoline is 6670 N/m.



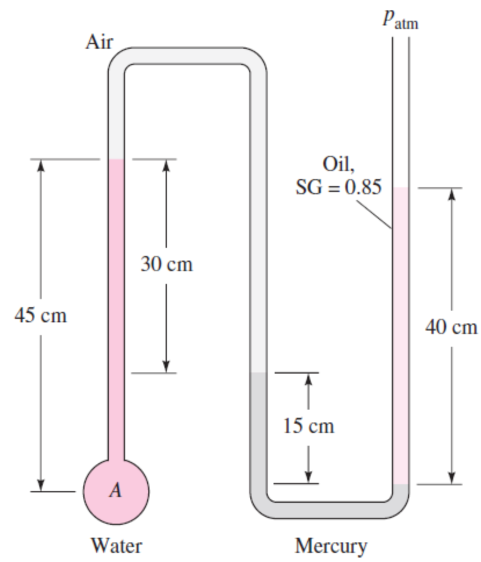
3. At 20C gage A reads 350 kPa absolute. What is the height h of the water in cm? What should gage B read in kPa absolute? Density of mercury is 13580 kg/m^3.

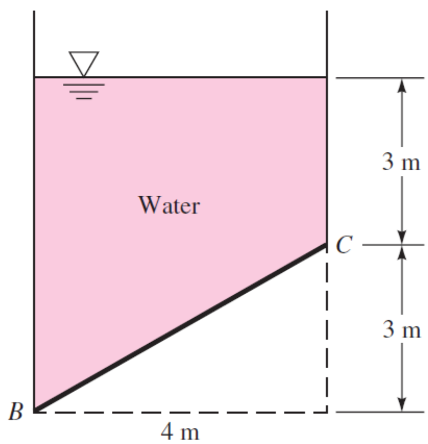


4. In the figure on the left, all fluids are at 20C. Determine the pressure difference between points A and B. Specific weights are:

Benzene:	8640 N/m^3	Mercury:	133100 N/m^3
Kerosene:	7885 N/m^3	Water:	9790 N/m^3

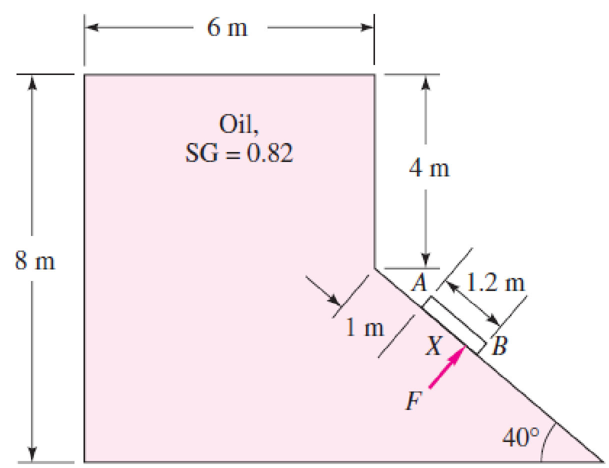
5. In the figure on the right, determine the gage pressure at point A. Is it higher or lower than atmospheric?

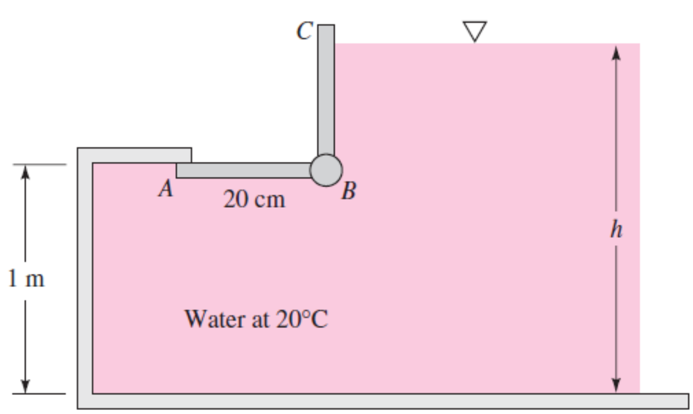




6. The tank in the figure on the left is 2 m wide into the paper. Neglecting atmospheric pressure, find the resultant hydrostatic force on the surface BC.

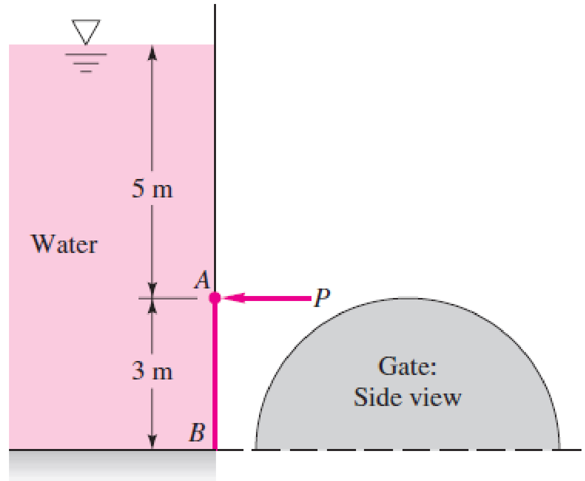
7. Gate AB in the figure on the right is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric pressure, compute the force F on the gate and its center-of-pressure position X.



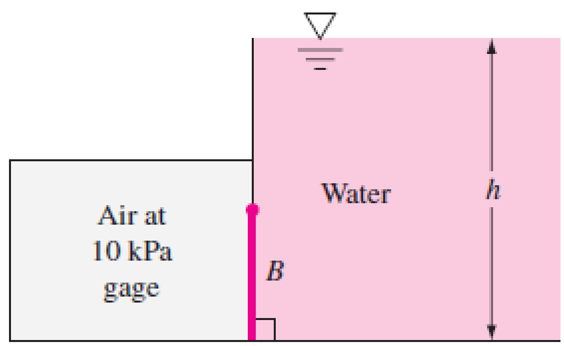


8. Gate ABC in the figure on the left has a fixed hinge line at B and is 2 m wide into the paper. The gate will open at A to release water if the water depth is high enough. Compute the depth h for which the gate will begin to open. Neglect the weight of the gate itself.

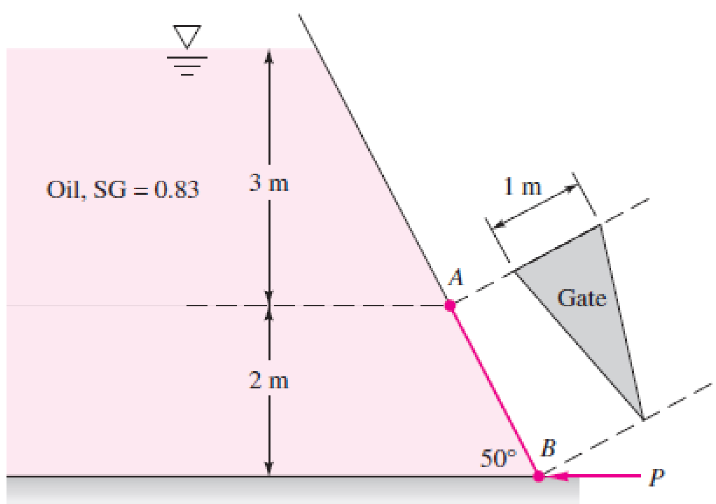
9. Gate AB in the Figure on the left is semicircular, hinged at B, and held by a horizontal force P at A. What force P is required for equilibrium?



10. Gate B in the figure below is 30 cm high, 60 cm wide into the paper, and hinged at the top. What water depth h will first cause the gate to open?



11. Isosceles triangle gate AB in the figure below is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?



Solution of assignment #2

1

2.12 In Fig. P2.12 the tank contains water and immiscible oil at 20°C. What is h in centimeters if the density of the oil is 898 kg/m³?

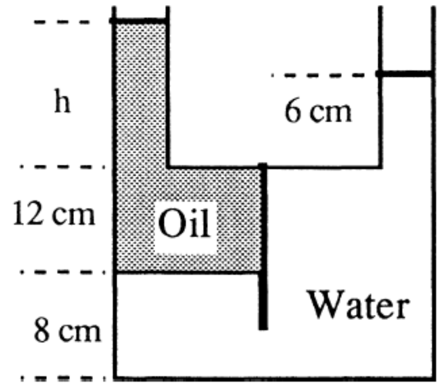


Fig. P2.12

Solution: For water take the density = 998 kg/m³. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

$$p_{atm} + (898)(g)(h + 0.12) - (998)(g)(0.06 + 0.12) = p_{atm},$$

Solve for $h \approx 0.08 \text{ m} \approx \mathbf{8.0\ cm}$ *Ans.*

2

2.13 In Fig. P2.13 the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid?

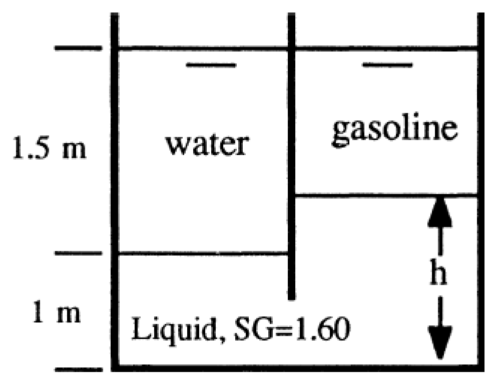


Fig. P2.13

Solution: Take water = 9790 N/m³ and gasoline = 6670 N/m³. The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

$$p_{bottom} = (9790 \text{ N/m}^3)(1.5 \text{ m}) + 1.60(9790)(1.0) = 1.60(9790)h + 6670(2.5 - h)$$

Solve for $h = \mathbf{1.52\ m}$ *Ans.*

3

2.21 In Fig. P2.21 all fluids are at 20°C. Gage A reads 350 kPa absolute. Determine (a) the height h in cm; and (b) the reading of gage B in kPa absolute.

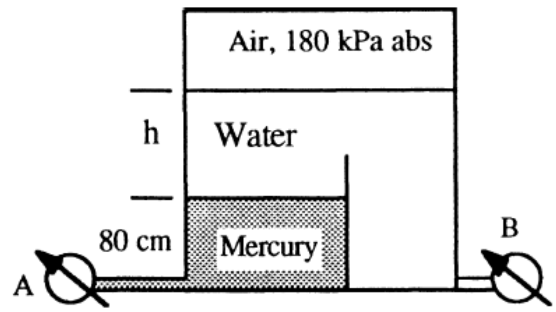


Fig. P2.21

Solution: Apply the hydrostatic formula from the air to gage A:

$$p_A = p_{air} + \sum \gamma h$$

$$= 180000 + (9790)h + 133100(0.8) = 350000 \text{ Pa},$$

Solve for $h \approx \mathbf{6.49\ m}$ *Ans.* (a)

Then, with *h* known, we can evaluate the pressure at gage B:

$$p_B = 180000 + 9790(6.49 + 0.80) = 251000 \text{ Pa} \approx \mathbf{251\ kPa} \quad \textit{Ans.}\ (b)$$

4

2.31 In Fig. P2.31 determine Δp between points A and B. All fluids are at 20°C.

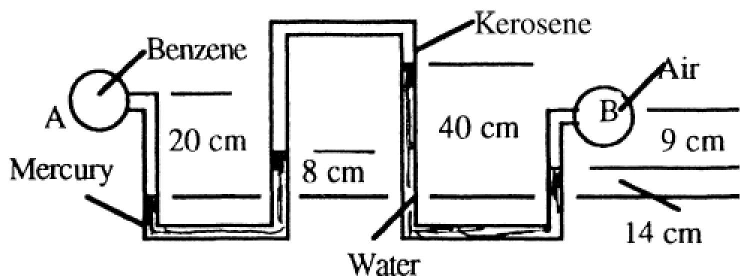


Fig. P2.31

Solution: Take the specific weights to be

Benzene: 8640 N/m^3 Mercury: 133100 N/m^3

Kerosene: 7885 N/m^3 Water: 9790 N/m^3

and γ_{air} will be small, probably around 12 N/m^3. Work your way around from A to B:

$$p_A + (8640)(0.20\text{ m}) - (133100)(0.08) - (7885)(0.32) + (9790)(0.26) - (12)(0.09)$$

$$= p_B, \quad \text{or, after cleaning up,} \quad p_A - p_B \approx \mathbf{8900\ Pa} \quad \textit{Ans.}$$

5

2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than $P_{atmosphere}$?

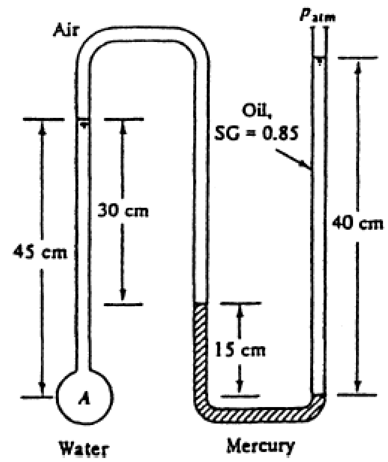


Fig. P2.45

Solution: Take $\gamma = 9790$ N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$p_{atm} + (0.85)(9790)(0.4\text{ m}) - (133100)(0.15\text{ m}) - (12)(0.30\text{ m}) + (9790)(0.45\text{ m}) = p_A,$$

$$\text{or:}\quad p_A = p_{atm} - 12200\text{ Pa} = \mathbf{12200\ Pa\ (vacuum)} \quad \textit{Ans.}$$

6

Solution: (a) The resultant force F, may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG} A = (9790\text{ N/m}^3)(3+1.5\text{ m})(5\text{ m}\times 2\text{ m}) = 440{,}550\text{ N} = \mathbf{441\ kN} \quad \textit{Ans.}\text{ (a)}$$

7

2.51 Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force F on the gate and its center of pressure position *X*.

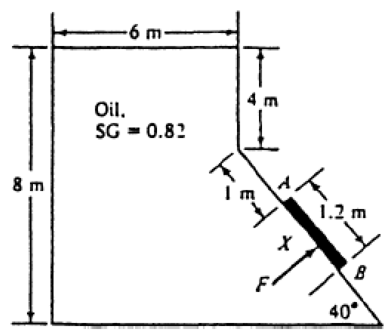


Fig. P2.51

Solution: The centroidal depth of the gate is

$$h_{CG} = 4.0 + (1.0 + 0.6)\sin 40° = 5.028\text{ m},$$

$$\text{hence}\quad F_{AB} = \gamma_{oil} h_{CG} A_{gate} = (0.82\times 9790)(5.028)(1.2\times 0.8) = \mathbf{38750\ N} \quad \textit{Ans.}$$

The line of action of F is slightly below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(0.8)(1.2)^3\sin 40°}{(5.028)(1.2\times 0.8)} = -0.0153\text{ m}$$

Thus the position of the center of pressure is at $X = 0.6 + 0.0153 \approx \mathbf{0.615\ m}$ *Ans.*

8

2.64 Gate ABC in Fig. P2.64 has a fixed hinge at B and is 2 m wide into the paper. If the water level is high enough, the gate will open. Compute the depth *h* for which this happens.

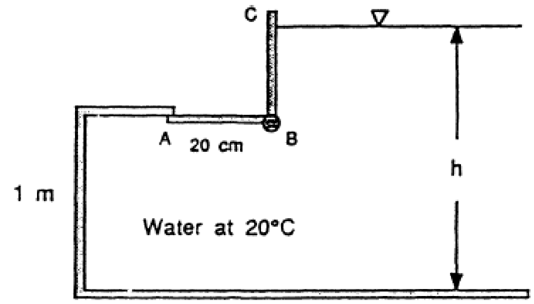
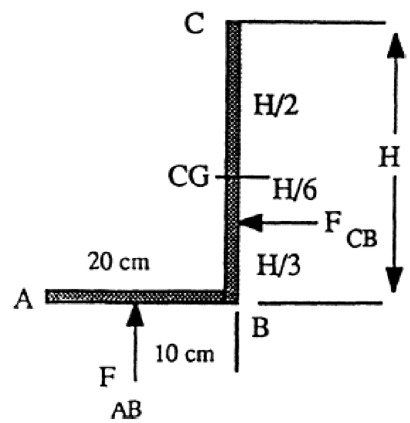


Fig. P2.64

Solution: Let *H* = (h − 1 meter) be the depth down to the level AB. The forces on AB and BC are shown in the freebody at right. The moments of these forces about B are equal when the gate opens:



$$\sum M_B = 0 = \gamma H(0.2)b(0.1)$$

$$= \gamma\left(\frac{H}{2}\right)(Hb)\left(\frac{H}{3}\right)$$

$$\text{or:}\quad H = 0.346 \text{ m},$$

$$h = H + 1 = \mathbf{1.346\ m} \quad \textit{Ans.}$$

This solution is independent of both the water density and the gate width *b* into the paper.

9

2.65 Gate AB in Fig. P2.65 is semi-circular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

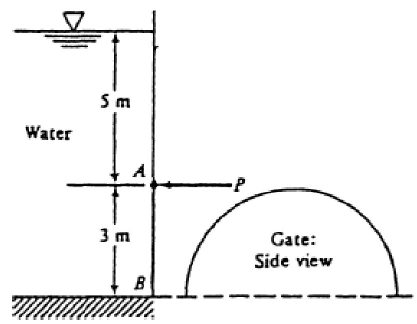
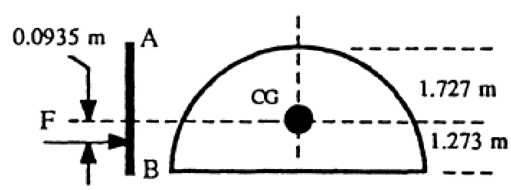


Fig. P2.65

Solution: The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is



$$F = \gamma h_{CG} A = (9790)(5.0 + 1.727)\frac{\pi}{2}(3)^2$$

$$= 931000 \text{ N}$$

The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(0.10976)(3)^4 \sin 90^\circ}{(5+1.727)(\pi/2)(3)^2} = -0.0935 \text{ m}$$

Then summing moments about B yields the proper support force P:

$$\sum M_B = 0 = (931000)(1.273 - 0.0935) - 3P, \quad \text{or:} \quad P = \mathbf{366000\ N} \quad \textit{Ans.}$$

10

2.72 Gate B is 30 cm high and 60 cm wide into the paper and hinged at the top. What is the water depth *h* which will first cause the gate to open?

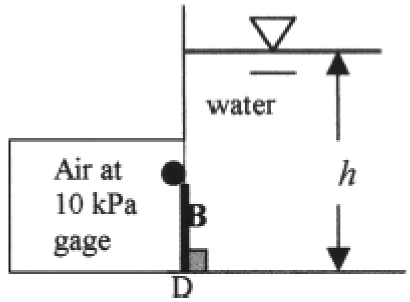


Fig. P2.72

Solution: The minimum height needed to open the gate can be assessed by calculating the hydrostatic force on each side of the gate and equating moments about the hinge. The air pressure causes a force, F_{air}, which acts on the gate at 0.15 m above point D.

$$F_{air} = (10{,}000\ \text{Pa})(0.3\ \text{m})(0.6\ \text{m}) = 1800\ \text{N}$$

Since the air pressure is uniform, F_{air} acts at the centroid of the gate, or 15 cm below the hinge. The force imparted by the water is simply the hydrostatic force,

$$F_w = (\gamma h_{CG} A)_w = (9790\ \text{N/m}^3)(h - 0.15\ \text{m})(0.3\ \text{m})(0.6\ \text{m}) = 1762.2h - 264.3$$

This force has a center of pressure at,

$$y_{CP} = \frac{\frac{1}{12}(0.6)(0.3)^3(\mathbf{sin90})}{(h-0.15)(0.3)(0.6)} = \frac{0.0075}{h-0.15} \quad \text{with } h \text{ in meters}$$

Sum moments about the hinge and set equal to zero to find the minimum height:

$$\sum M_{hinge} = 0 = (1762.2h - 264.3)[0.15 + (0.0075/(h-0.15))] - (1800)(0.15)$$

This is quadratic in *h*, but let's simply solve by iteration: $h = \mathbf{1.12\ m}$ *Ans.*

11

2.68 Isosceles triangle gate AB in Fig. P2.68 is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?

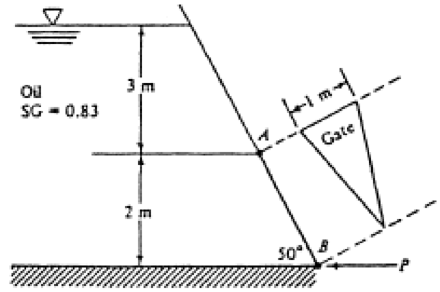
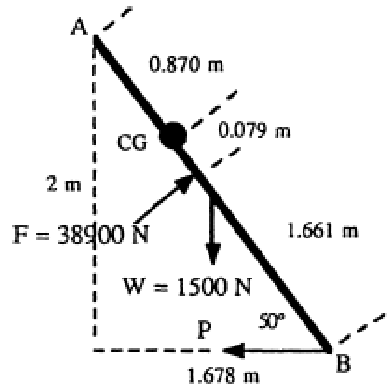


Fig. P2.68

Solution: The gate is 2.0/sin 50° = 2.611 m long from A to B and its area is 1.3054 m^2. Its centroid is 1/3 of the way down from A, so the centroidal depth is 3.0 + 0.667 m. The force on the gate is

$$F = \gamma h_{CG} A = (0.83)(9790)(3.667)(1.3054)$$
$$= 38894 \text{ N}$$



The position of this force is below the centroid:

$$y_{CP} = -\frac{I_{xx} \sin\theta}{h_{CG} A}$$

$$= -\frac{(1/36)(1.0)(2.611)^3 \sin 50°}{(3.667)(1.3054)} = -0.0791 \text{ m}$$

The force and its position are shown in the freebody at upper right. The gate weight of 1500 N is assumed at the centroid of the plate, with moment arm 0.559 meters about point A. Summing moments about point A gives the required force P:

$$\sum M_A = 0 = P(2.0) + 1500(0.559) - 38894(0.870 + 0.0791),$$

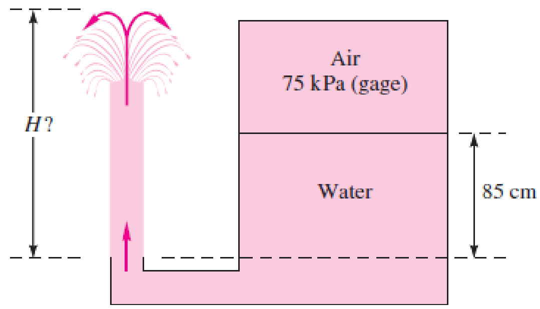
Solve for $P = \mathbf{18040\ N}$ *Ans.*

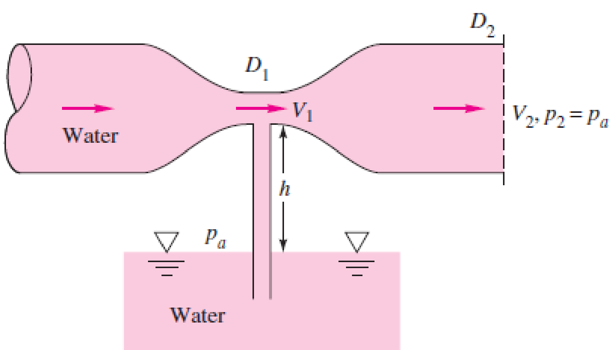
M201 Hydraulics

Assignment #3: Bernoulli Equation

1. What pressure gradient along the streamline, *dP/ds* , is required to accelerate water in a horizontal pipe at a rate of 30 m/s^2?

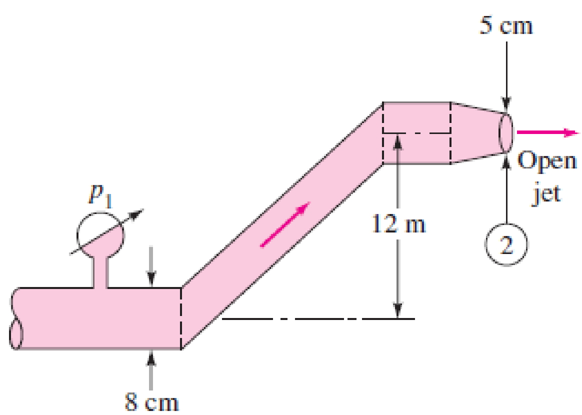
2. Water at 20°C, in the pressurized tank (figure on the right) flows out and creates a vertical jet as shown. Assuming steady frictionless flow, determine the height H to which the jet rises.

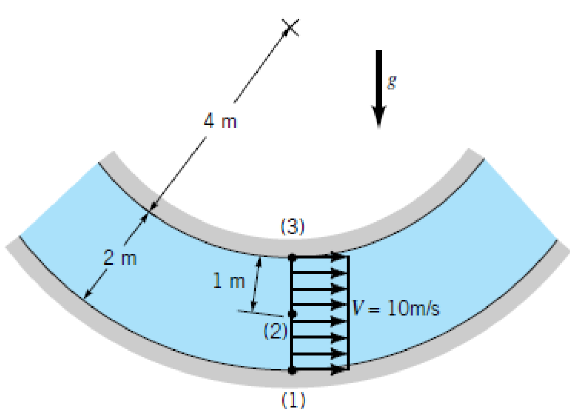




3. A necked-down section in a pipe flow, called a venturi (figure on the left), develops a low throat pressure that can aspirate fluid upward from a reservoir. Derive an expression for the velocity V_1 that is just sufficient to bring reservoir fluid into the throat.

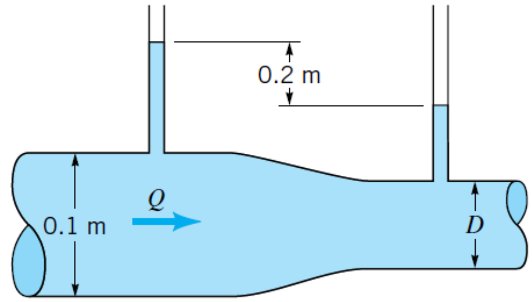
4. In the figure on the right, the fluid is water at a weight flux of 120 N/s. Assuming no losses, estimate the gage pressure at section 1.



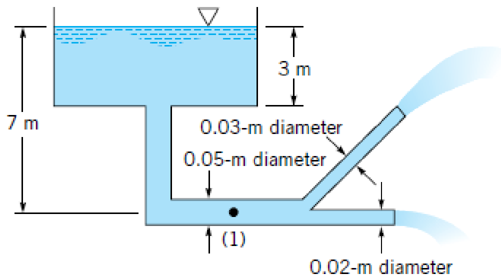


5. Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in the figure on the left. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

6. Water flows through the pipe contraction shown in the figure. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.



7. Water flows from a large tank through a large pipe that splits into two smaller pipes as shown in the figure on the right. If viscous effects are negligible, determine the flowrate from the tank and the pressure at point (1).



Solutions of Assignment # 3

(1) $$\frac{\partial P}{\partial s} + \gamma \frac{\partial z}{\partial s} + \rho V \frac{\partial V}{\partial s} = 0$$

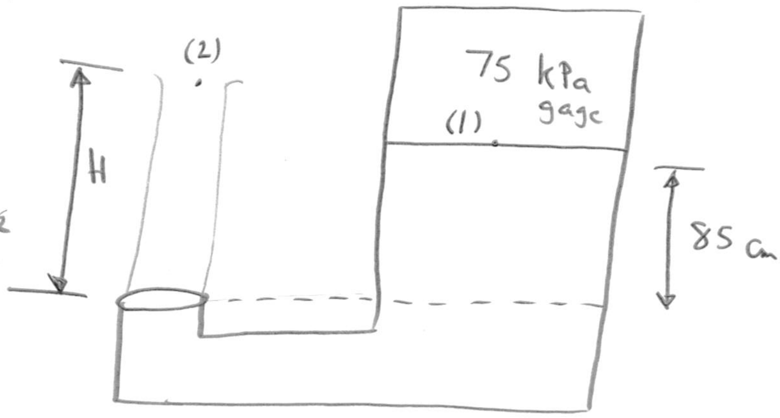
horizontal plane $\Longrightarrow \frac{\partial z}{\partial s} = 0$

$$\therefore \frac{\partial P}{\partial s} = -\rho V \frac{\partial V}{\partial s}$$

$$= -\rho a_s = -1000 \times 30 = \boxed{-30 \text{ kPa/m}}$$

---

(2) Apply Bernoulli eqn between points (1) & (2);



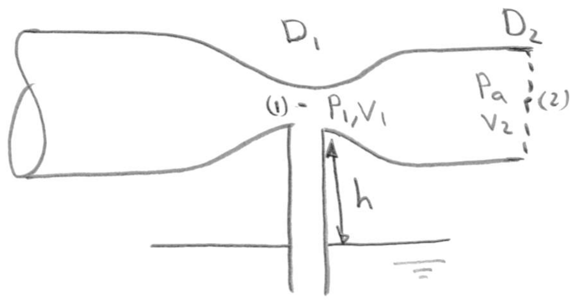
$$P_1 + \gamma z_1 + \cancel{\tfrac{1}{2}\rho V_1^2}^{\,0} = \cancel{P_2}^{\,0} + \gamma z_2 + \cancel{\tfrac{1}{2}\rho V_2^2}^{\,0}$$

$$\therefore 75 \times 10^3 + 9800 \times 0.85 = 9800\, H$$

$$\therefore \boxed{H = 8.5 \text{ m}}$$

---

(3) $$\boxed{P_1 = P_a - \gamma h}$$



Apply Bernoulli eqn between (1) & (2)

$$P_1 + \cancel{\gamma z_1} + \tfrac{1}{2}\rho V_1^2 = P_2 + \cancel{\gamma z_2} + \tfrac{1}{2}\rho V_2^2$$

$$\therefore \cancel{P_a} - \gamma h + \tfrac{1}{2}\rho V_1^2 = \cancel{P_a} + \tfrac{1}{2}\rho V_2^2$$

$$h = \tfrac{1}{2}\rho (V_1^2 - V_2^2)\,\frac{1}{\gamma} = \frac{1}{2g}(V_1^2 - V_2^2)$$

but $A_1 V_1 = A_2 V_2 \quad \therefore D_1^2 V_1 = D_2^2 V_2$

$$\therefore h = \frac{1}{2g}\left[V_1^2 - \left(\frac{D_1}{D_2}\right)^4 V_1^2\right] = \frac{V_1^2}{2g}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

$$\therefore \boxed{V_1 = \sqrt{\frac{2gh}{1-\left(\frac{D_1}{D_2}\right)^4}}}$$

4 Weight flux = 120 N/s

$\therefore \rho g V A = 120$

$\therefore 9800 \times V_2 \times \frac{\pi}{4}(0.05)^2 = 120$

$\therefore \boxed{V_2 = 6.24 \text{ m/s}}$, similarly $\boxed{V_1 = 2.44 \text{ m/s}}$

Apply Bernoulli eqn along a stream line from point (1) to point (2)

$$P_1 + \cancel{\gamma z_1}^{0} + \frac{1}{2}\rho V_1^2 = \cancel{P_2}^{0} + \gamma z_2 + \frac{1}{2}\rho V_2^2$$

$$\therefore P_1 + 0 + \frac{1}{2} \times 1000 \times (2.44)^2 = 9800 \times 12 + \frac{1}{2} \times 1000 \times (6.24)^2$$

$$\therefore \boxed{P_1 = 134.09 \text{ kPa}}$$

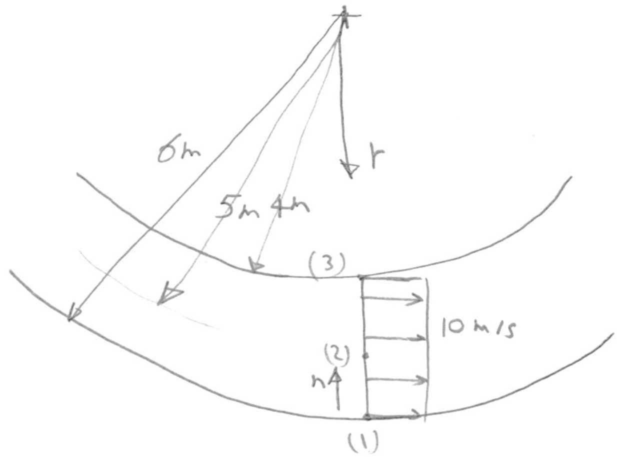
(3)

5) Newton's second law across stream lines:

$$P + \gamma z + \int \rho \frac{V^2}{R} dn = 0$$

or:

$$\frac{dP}{dn} + \gamma \frac{dz}{dn} + \rho \frac{V^2}{R} = 0$$



$R =$

$$r = 6 - n$$

$$dr = -dn = -dz$$

$$\therefore \quad -\frac{dP}{dr} + \gamma + \rho \frac{V^2}{r} = 0$$

$$\therefore \int \frac{dP}{dr} dr = \int \gamma \, dr + \int \rho \frac{V^2}{r} dr$$

integrate from point (1) to point (2)

$$P_2 - P_1 = \gamma (r_2 - r_1) + \rho V^2 \ln \frac{r_2}{r_1}$$

$$\therefore P_2 = 40 \times 10^3 + 9800 (5 - 6) + 1000 \times 10^2 \ln \frac{5}{6}$$

$$= \boxed{11.97 \text{ kPa}}$$

integrate from point (1) to point (3):

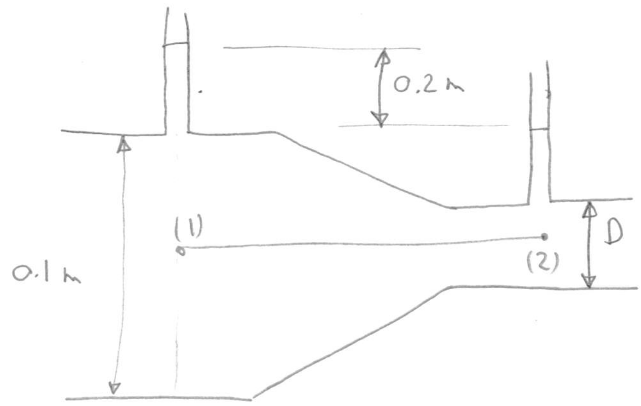
$$P_3 - P_1 = \gamma (r_3 - r_1) + \rho V^2 \ln \frac{r_3}{r_1}$$

$$P_3 = 40 \times 10^3 + 9800 (4 - 6) + 1000 \times 10^2 \ln \frac{4}{6}$$

$$= \boxed{-20.15 \text{ kPa}}$$

(6) (4)

$$P_1 + \cancel{\gamma} z_1 + \frac{1}{2}\rho V_1^2 = P_2 + \cancel{\gamma} z_2 + \frac{1}{2}\rho V_2^2$$



$$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

$$V_1 = \frac{Q}{\frac{\pi}{4}D_1^2} = \frac{Q}{\frac{\pi}{4}(0.1)^2} = 127.3\,Q$$

$$V_2 = \frac{Q}{\pi/4\,D^2} = 1.27\,\frac{Q}{D^2}$$

$\therefore P_1 - P_2 = 9800 \times 0.2 = 1960$ Pa

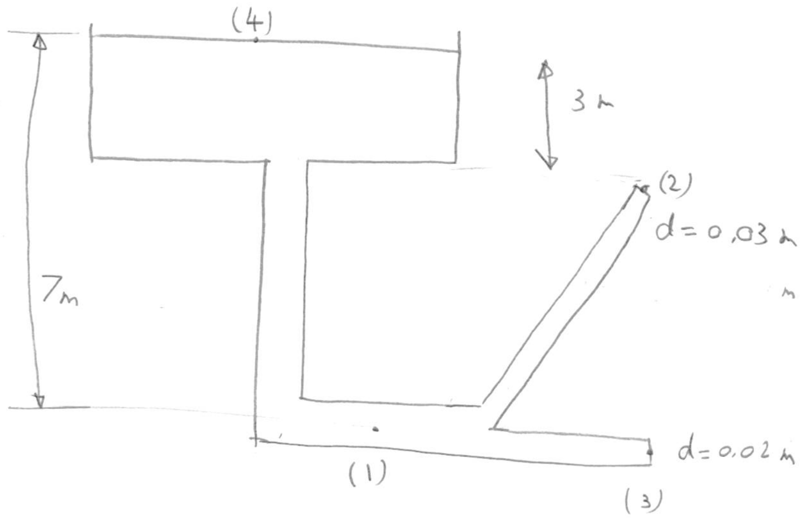
$$\therefore 1960 = \frac{1}{2} \times 1000 \left(1.27^2 \frac{Q^2}{D^4} - 127.3^2 Q^2\right)$$

$$3.92 = 1.613\,\frac{Q^2}{D^4} - 16205.3\,Q^2$$

$$\therefore \boxed{D^4 = \frac{1.613\,Q^2}{3.92 + 16205.3\,Q^2}}$$

7) Apply Bernoulli between (4) & (2)

5



$$\therefore \cancel{P_4} + \gamma Z_4 + \cancel{\tfrac{1}{2}\rho V_4^2} = \cancel{P_2} + \gamma Z_2 + \tfrac{1}{2}\rho V_2^2$$

$$\therefore 9800 \times 7 = 9800 \times 4 + \tfrac{1}{2} \times 1000 \times V_2^2$$

$$\therefore V_2 = \boxed{7.67 \text{ m/s}}$$

* apply Bernoulli between (4) & (3)

$$\cancel{P_4} + \gamma Z_4 + \cancel{\tfrac{1}{2}\rho V_4^2} = \cancel{P_3} + \cancel{\gamma Z_3} + \tfrac{1}{2}\rho V_3^2$$

$$\therefore 9800 \times 7 = \tfrac{1}{2} \times 1000 \times V_3^2$$

$$\therefore V_3 = \boxed{11.71 \text{ m/s}}$$

$$\therefore Q_T = Q_2 + Q_3$$

$$= 7.67 \times \tfrac{\pi}{4}(0.03)^2 + 11.71 \times \tfrac{\pi}{4}(0.02)^2 = 9.1 \times 10^{-3} \text{ m}^3/\text{s}$$

$$= \boxed{9.1 \text{ Litre/s}}$$

M201 Hydraulics

Assignment #4: Bernoulli Equation

1. Water (assumed inviscid and incompressible) flows steadily in the vertical variable-area pipe shown in Fig. P3.45. Determine the flowrate if the pressure in each of the gages reads 50 kPa.

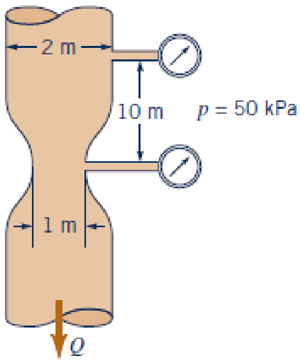


Figure P3.45

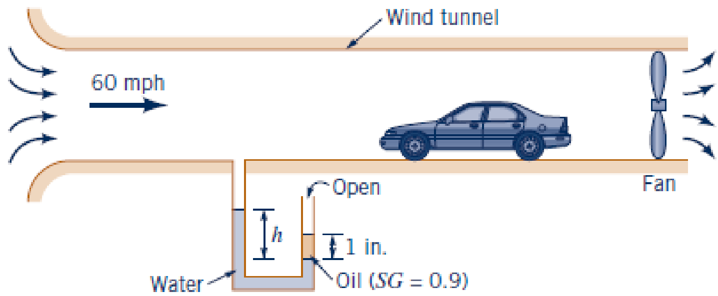


Figure P3.46

2. Air is drawn into a wind tunnel used for testing automobiles as shown in Fig. P3.46. (a) Determine the manometer reading, h, when the velocity in the test section is 60 mph. Note that there is a 1-in. column of oil on the water in the manometer. (b) Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.

3. Water flows through the pipe contraction shown in Fig. P3.52. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

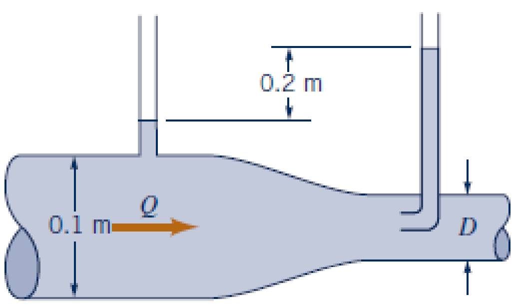


Figure P3.52

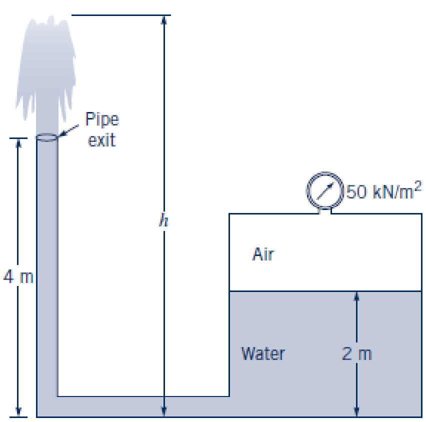
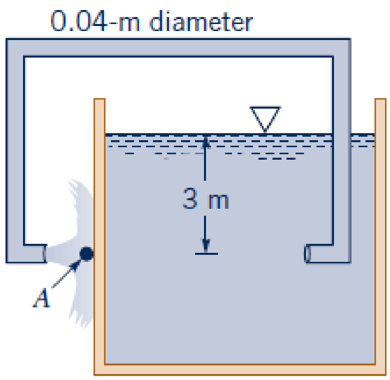


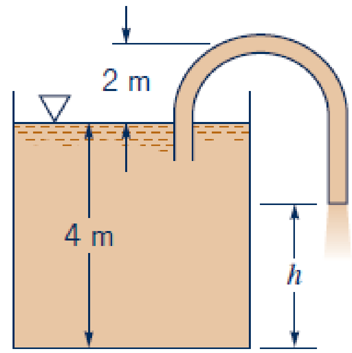
Figure P3.49

4. Water (assumed frictionless and incompressible) flows steadily from a large tank and exits through a vertical, constant diameter pipe as shown in Fig. P3.49. The air in the tank is pressurized to 50 kN/m^2. Determine (a) the height h, to which the water rises, (b) the water velocity in the pipe, and (c) the pressure in the horizontal part of the pipe.

5. Water is siphoned from a tank as shown in Fig. P3.59. Determine the flowrate and the pressure at point A, a stagnation point.



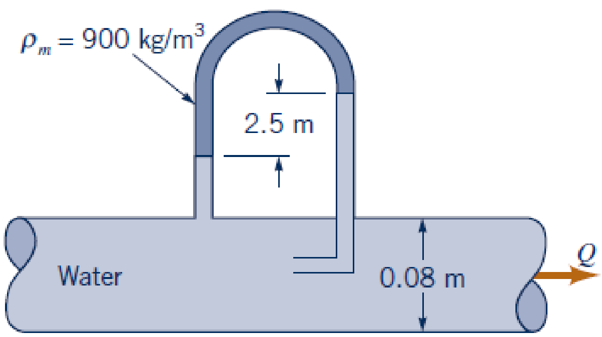
■ Figure P3.59



■ Figure P3.60

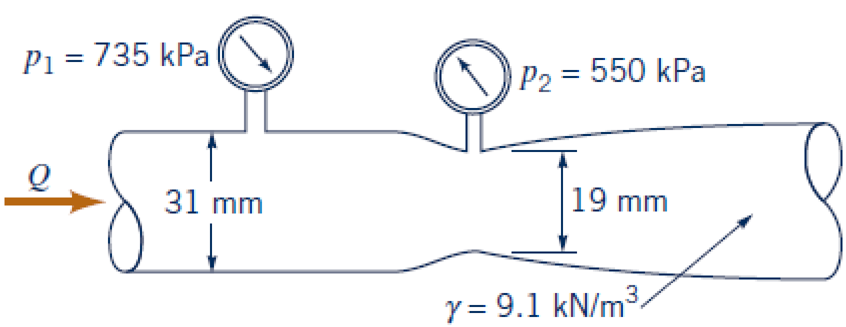
6. A 50-mm-diameter plastic tube is used to siphon water from the large tank shown in Fig. P3.60. If the pressure on the outside of the tube is more than 30 kPa greater than the pressure within the tube, the tube will collapse and siphon will stop. If viscous effects are negligible, determine the minimum value of h allowed without the siphon stopping.

7. Determine the flowrate through the pipe in Fig. P3.69.

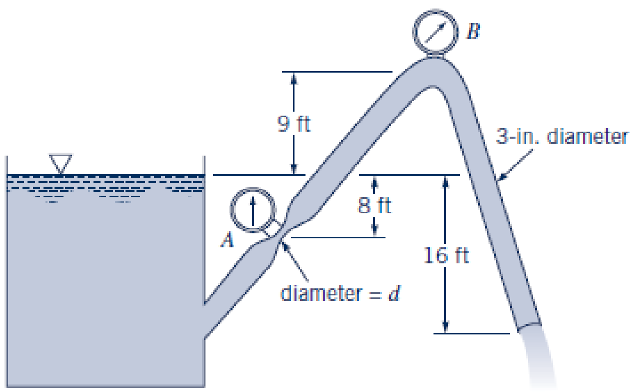


■ Figure P3.69

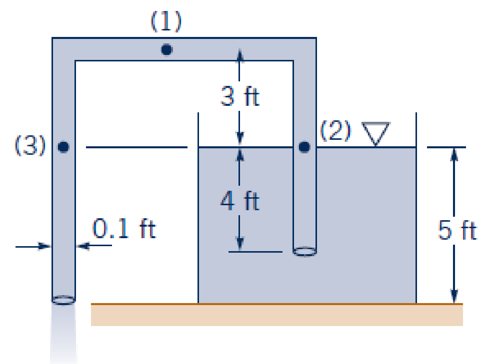
8. Determine the flowrate through the Venturi meter shown in the figure below if ideal conditions exist.



9. Water flows steadily from a large open tank and discharges into the atmosphere though a 3-in.-diameter pipe as shown in Fig. P3.82. Determine the diameter, d, in the narrowed section of the pipe at A if the pressure gages at A and B indicate the same pressure.



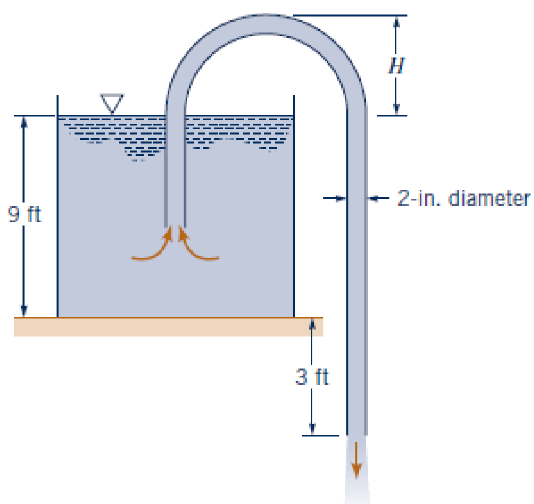
■ Figure P3.82



■ Figure P3.87

10. Water is siphoned from the tank shown in Fig. P3.87. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.

11. Water is siphoned from a large tank and discharges into the atmosphere through a 2-in.-diameter tube as shown in Fig. P3.88. The end of the tube is 3 ft below the tank bottom, and viscous effects are negligible. (a) Determine the volume flowrate from the tank. (b) Determine the maximum height, H, over which the water can be siphoned without cavitation occurring. Atmospheric pressure is 14.7 psia, and the water vapor pressure is 0.26 psia.

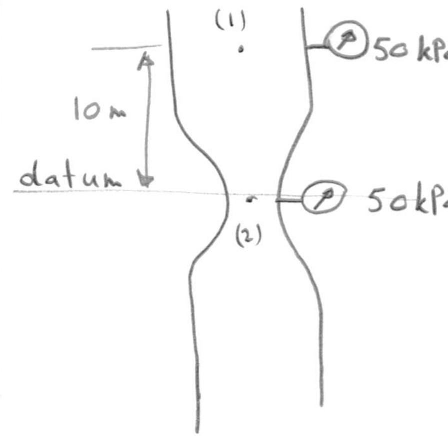


■ Figure P3.88

Solution of Assignment #4

① $V_1 A_1 = V_2 A_2$

$\therefore V_1 = V_2 \left(\frac{1}{2}\right)^2 = \frac{1}{4} V_2$



Apply Bernoulli between ① & ②:

$$\cancel{P_1} + \gamma z_1 + \frac{1}{2}\rho V_1^2 = \cancel{P_2} + \cancel{\gamma z_2}^{\,0} + \frac{1}{2}\rho V_2^2$$

$$\therefore 9800 \times 10 = \frac{1}{2} \times 1000 \left(V_2^2 - \left(\frac{1}{4}\right)^2 V_2^2 \right)$$

$$\therefore 196 = \frac{15}{16} V_2^2$$

$$\therefore V_2 = 14.46 \text{ m/s}$$

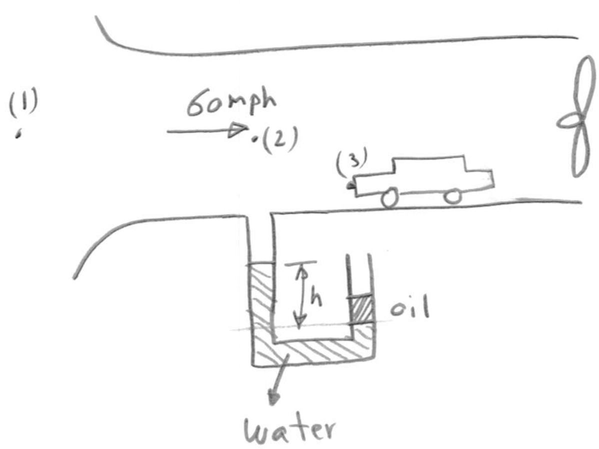
$$\therefore Q = 14.46 \times \frac{\pi}{4}(1)^2 = \boxed{11.36 \text{ m}^3/\text{s}}$$

---

② $V_2 = 60$ mph

$$= \frac{60 \text{ mile}}{\text{hr}} \left| \frac{1609 \text{ m}}{1 \text{ mile}} \right| \frac{1 \text{ hr}}{3600 \text{ sec.}}$$

$$= 26.82 \text{ m/s}$$



Bernoulli (1) ⟶ (2)

$$\cancel{\frac{P_1}{\gamma}}^{\,0} + \cancel{z_1} + \cancel{\frac{V_1^2}{2g}}^{\,0} = \frac{P_2}{\gamma} + \cancel{z_2} + \frac{V_2^2}{2g}$$

$$\frac{P_2}{\gamma} = -\frac{V_2^2}{2g}$$

air density

$$\therefore P_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2} \times 1.26 \times (26.82)^2$$

$$= \boxed{-453.1 \text{ Pa}}$$

In the manometer:

$$\cancel{P_a}^{\,0} + \gamma_{oil} \times 0.0254 = P_2 + 9800h$$

$$\therefore 0.9 \times 9800 \times 0.0254 - (-453.1) = 9800h$$

$$\therefore h = 0.069 \text{ m} = \boxed{6.9 \text{ cm}}$$

$P_3 = P_{stagnation}$

$$= P_2 + \frac{1}{2}\rho V_2^2$$

$$= -453.1 + \frac{1}{2} \times 1.26 \times 26.82^2$$

$$= \text{Zero}$$

(2)

(3) $P_1 + \frac{1}{2}\rho V_1^2 = P_2$

but $P_2 - P_1 = \gamma \times 0.2$

$\therefore 0.2\,\gamma = \frac{1}{2}\rho V_1^2$

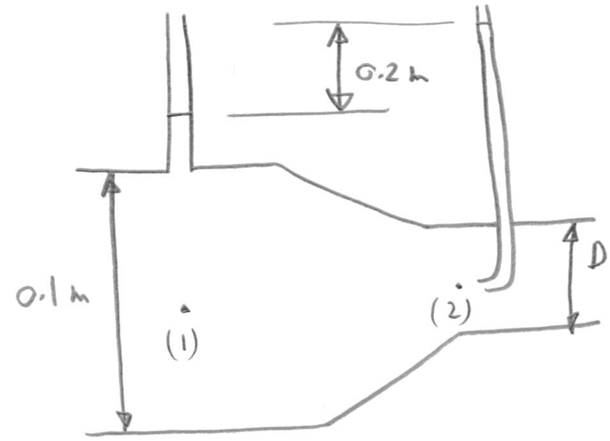
$\therefore V_1 = \sqrt{\dfrac{2 \times 0.2 \times 9800}{1000}}$

$= \boxed{1.98\ m/s}$

$\therefore Q = V_1 A_1$

$= 1.98 \times \frac{\pi}{4}(0.1)^2 = \boxed{0.0156\ m^3/s}$

Q does not depend on D



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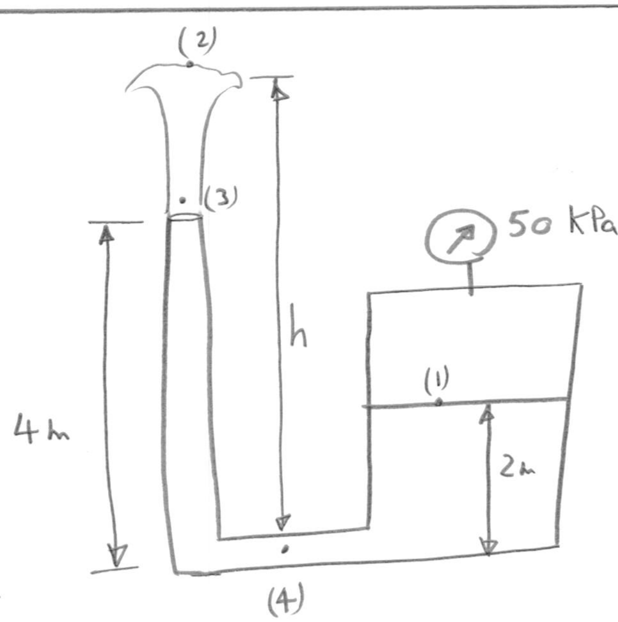
4) Bernoulli (1) & (2)

$P_1 + \gamma z_1 + \cancel{\frac{1}{2}\rho V_1^2} = \cancel{P_2} + \gamma z_2 + \cancel{\frac{1}{2}\rho V_2^2}$

$\therefore 50 \times 10^3 + 9800 \times 2 = 9800 \times h$

$\therefore h = \boxed{7.1\ m}$



* Bernoulli (1) → (3)

$P_1 + \gamma z_1 + \cancel{\frac{1}{2}\rho V_1^2} = \cancel{P_3} + \gamma z_3 + \frac{1}{2}\rho V_3^2$

$\therefore 50 \times 10^3 + 9800 \times 2 = 9800 \times 4 + 500\, V_3^2$

$\therefore V_3 = \boxed{7.8\ m/s}$

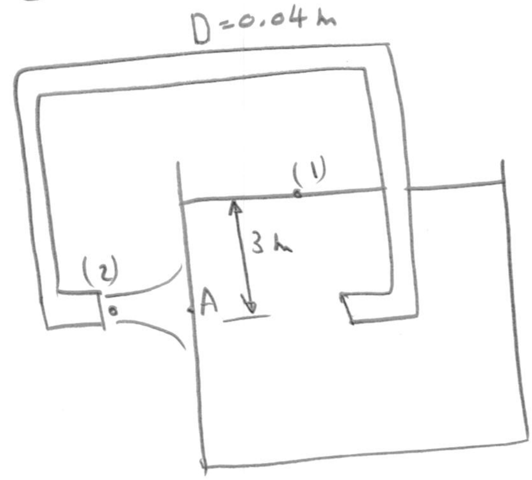
* Bernoulli (1) → (4)

$V_4 = V_3 = 7.8\ m/s$

$50 \times 10^3 + 9800 \times 2 = P_4 + \cancel{\gamma z_4} + \frac{1}{2}\rho V_4^2$

$\therefore P_4 = \boxed{39.18\ kPa}$

5 Bernoulli (1) $\longrightarrow$ (2)



$$\gamma z_1 = \frac{1}{2}\rho V_2^2$$

$$\therefore V_2 = \sqrt{2 g z_1}$$

$$= \sqrt{2 \times 9.8 \times 3} = \boxed{7.67 \text{ m/s}}$$

Bernoulli (2) $\longrightarrow$ (A)

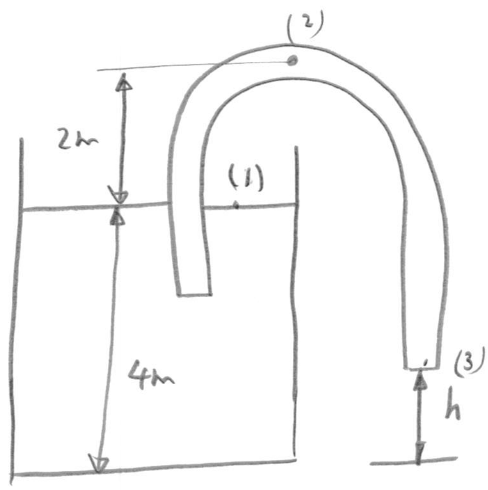
$$\frac{1}{2}\rho V_2^2 = P_A$$

$$P_A = \frac{1}{2} \times 1000 \times (7.67)^2 = \boxed{29.4 \text{ kPa}}$$

---

6 Minimum pressure exists at (2)

$$\therefore P_2 = -30 \text{ kPa}$$



Bernoulli (1) $\longrightarrow$ (2)

$$\overset{0}{\cancel{P_1}} + \gamma z_1 + \frac{1}{2}\overset{0}{\cancel{\rho V_1^2}} = P_2 + \gamma z_2 + \frac{1}{2}\rho V_2^2$$

$$9800 \times 4 = -30 \times 10^3 + 9800 \times 6 + \frac{1}{2} \times 1000 V_2^2$$

$$\therefore V_2 = 4.56 \text{ m/s}$$

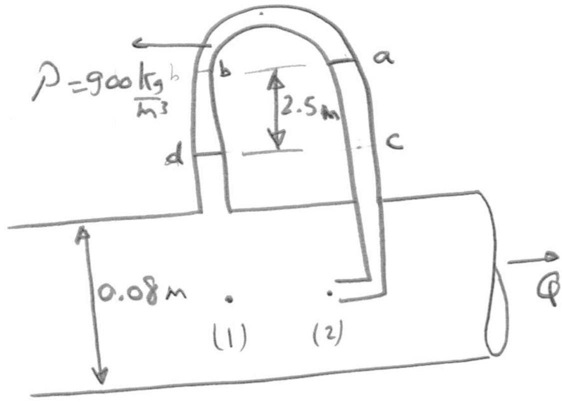
Bernoulli (1) $\longrightarrow$ (3)

$$\gamma z_1 = \gamma h + \frac{1}{2}\rho V_3^2$$

$$\therefore 9800 \times 4 = 9800 h + \frac{1}{2} \times 1000 \times (4.56)^2$$

$$\therefore \boxed{h = 2.94 \text{ m}}$$

7 Bernoulli $(1) \longrightarrow (2)$



$$P_1 + \frac{1}{2}\rho V_1^2 = P_2$$

$$\therefore V_1 = \sqrt{\frac{2}{\rho}(P_2 - P_1)}$$

but $P_2 - P_1 = 2.5(\gamma_w - \gamma_{oil})$

$$= 2.5\ (9800 - 9.8 \times 900)$$

$$= 2450 \text{ Pa}$$

$$\therefore V_1 = \sqrt{\frac{2}{1000}(2450)} = \boxed{2.21 \text{ m/s}}$$

$$Q = \frac{\pi}{4} \times 0.08^2 \times 2.21 = \boxed{0.011 \text{ m}^3/\text{s}}$$

$$P_c = P_a + 2.5\gamma_w$$

$$P_d = P_b + 2.5\gamma_{oil}$$

$$\therefore P_c - P_d = (P_a - P_b) + 2.5(\gamma_w - \gamma_{oil})$$

but $P_a = P_b$

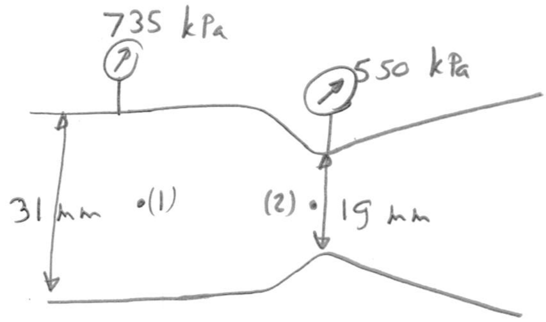
$$\therefore P_c - P_d = 2.5(\gamma_w - \gamma_{oil})$$

$$\therefore P_2 - P_1 = P_c - P_d$$

$$\therefore P_2 - P_1 = 2.5(\gamma_w - \gamma_{oil})$$

---

8 Bernoulli $(1) \longrightarrow (2)$



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{735 \times 10^3}{9100} + \frac{V_1^2}{19.6} = \frac{550 \times 10^3}{9100} + \frac{V_2^2}{19.6}$$

$$\therefore 20.32 + \frac{(0.376)^2 V_2^2}{19.6} = \frac{V_2^2}{19.6}$$

$$\therefore V_2 = 21.54 \text{ m/s}$$

$$Q = 21.54 \times \frac{\pi}{4}(0.019)^2$$

$$= \boxed{6.108 \times 10^{-3} \text{ m}^3/\text{s}}$$

from continuity:

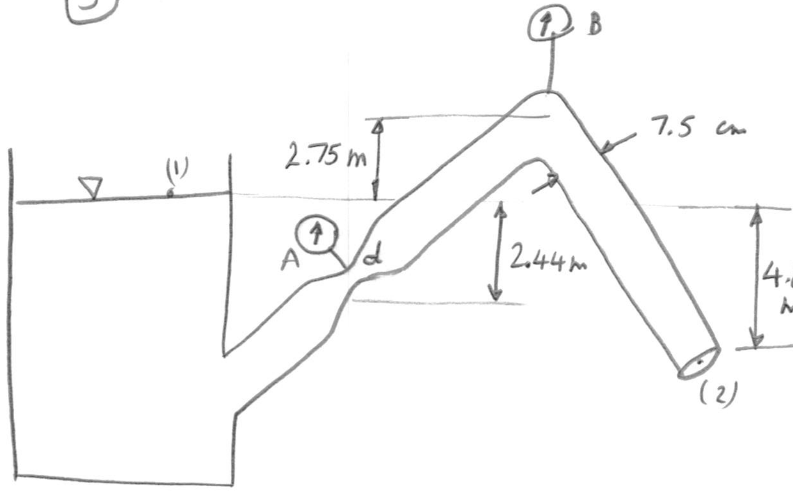
$$V_1 A_1 = V_2 A_2$$

$$\therefore V_1 = \left(\frac{19}{31}\right)^2 V_2$$

$$= 0.376\, V_2$$

5

9 Bernoulli (1) $\rightarrow$ (2)



$$\gamma z_1 = \frac{1}{2}\rho V_2^2$$

$$\therefore 9800 \times 4.88 = \frac{1000}{2} V_2^2$$

$$\therefore \boxed{V_2 = 9.78 \text{ m/s}}$$

$$V_A A_A = V_B A_B$$

$$\therefore V_A \frac{\pi}{4} d^2 = 9.78 \frac{\pi}{4}(0.075)^2$$

$$\therefore \boxed{V_A = \frac{0.055}{d^2}}$$

Bernoulli (A) $\rightarrow$ (B)

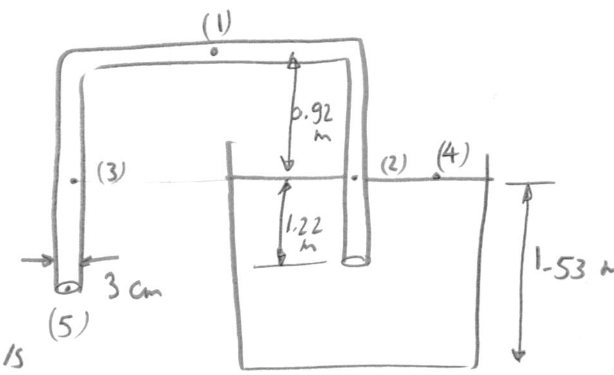
$$\cancel{P_A} + \gamma z_A + \frac{1}{2}\rho V_A^2 = \cancel{P_B} + \gamma z_B + \frac{1}{2}\rho V_B^2$$

$$\therefore 9800 \times 2.44 + \frac{1}{2}\times 1000\left(\frac{0.055}{d^2}\right)^2 = 9800 \times 7.63 + \frac{1}{2}\times 1000 \times (9.78)^2$$

$$\therefore \boxed{d = 0.063 \text{ m}}$$

---

10 Bernoulli (4) $\rightarrow$ (5)



$$V_5 = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \times 1.53} = 5.48 \text{ m/s}$$

$$Q = VA = \frac{\pi}{4}(0.03)^2 \times 5.48 = 3.87 \times 10^{-3} \text{ m}^3/\text{s}$$

Bernoulli (4) $\rightarrow$ (2) :

$$\gamma \cancel{z_4} = \cancel{\gamma z_2} + P_2 + \frac{1}{2}\rho V_2^2$$

$$\therefore P_2 = -\frac{1}{2}\rho V_2^2 = -500 \times (5.48)^2 = \boxed{-15.02 \text{ kPa}}$$

$\therefore P_3 = -15.02$ kPa because its at the same elevation like (2) and have the same velocity

$$P_1 = P_2 - \gamma(z_1 - z_2) = -15.02 \times 10^3 - 9800(0.92) = \boxed{-24.03 \text{ kPa}}$$

(11) Bernoulli (1) → (2) :

$V_2 = \sqrt{2gh}$

$= \sqrt{2 \times 9.8 \times (2.745 + 0.92)}$

$= \boxed{8.48 \text{ m/s}}$

Bernoulli (1) → (3) :

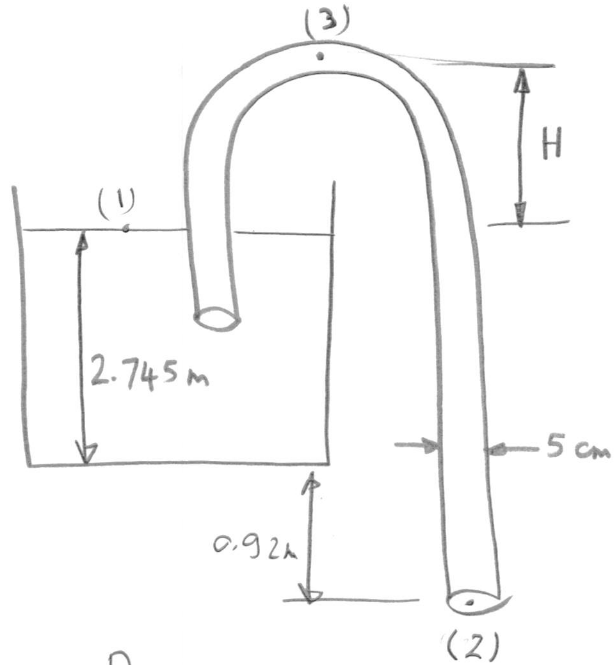
$\gamma z_1 = P_3 + \gamma z_3 + \frac{1}{2} \rho V_3^2$

$\therefore \cancel{9800 \times (2.745 + 0.92)} = -99.51 \times 10^3$

$+ 9800 (\cancel{2.745 + 0.92} + H) + \frac{1000}{2} (8.48)^2$

$\therefore \boxed{H = 6.485 \text{ m}}$

(6)



$P_a = 14.7 \text{ Psia}$
$= 101.3 \text{ kPa}$

$P_v = 0.26 \text{ Psia}$
$= 1.79 \text{ kPa}$

$P_{v \, gage} = 1.79 - 101.3 = -99.51 \text{ k}$